

Chapter 2

Le Pen, Nader, and Other Inconveniences

Focus Questions

In this chapter, we'll explore the following questions:

- What is the plurality method for determining the winner of an election? How does it differ from majority rule?
- What is the Borda count? How is it defined, and where is it commonly used?
- What is the majority criterion? Is it satisfied by plurality? By the Borda count?
- How do the plurality and Borda count methods relate to May's Theorem?

Warmup 2.1. The popular vote totals from the state of New Hampshire in the 2016 U.S. presidential election are given in Table 2.1.

- (a) In this election, did any of the candidates receive a *majority* (i.e., more than half) of the popular votes cast in the state of New Hampshire?
- (b) If Hillary Clinton and Donald Trump had been the only candidates in the 2016 U.S. presidential election, do you think that Trump might have possibly received more popular votes than Clinton in New Hampshire?

As you may recall, the 2016 U.S. presidential election was hotly contested, and the wounds from a bruising campaign season were not quick to heal. Many battleground states, like New Hampshire, were remarkably close.

Candidate	Popular Votes
Hillary Clinton	348,526
Donald Trump	345,790
Gary Johnson	30,777
Others	19,203

TABLE 2.1. 2016 U.S. presidential election in New Hampshire

As a percentage of the votes cast in the state, the difference between the vote totals of the two main candidates was a minuscule 0.368%. Michigan, which was won by Trump, was even closer at 0.223%.

Both of these differences, however, pale in comparison to the microscopic 0.009% difference between the vote totals in Florida for the two main candidates in the 2000 U.S. presidential election (shown in Table 2.2).¹

Candidate	Popular Votes
George W. Bush	2,912,790
Al Gore	2,912,253
Ralph Nader	97,488
Others	40,579

TABLE 2.2. 2000 U.S. presidential election in Florida

To make matters worse, counting and recounting of the popular votes in Florida delayed final certification of the results until over a month after the ballots were cast. And unlike 2016, when no single competitive state alone swung the outcome of the national election, Florida was the deciding factor in the 2000 election. In other words, the winner in Florida would have won the national election, whether it had been George W. Bush or Al Gore. Ultimately, Bush won the state by a razor-thin margin, resulting in him winning the national election and becoming the 43rd president of the United States.

Many political scientists speculate that if Ralph Nader had not been a candidate, Gore would have won Florida and, consequently, the national

¹It's important to note here that U.S. presidential elections are not decided by simply declaring the winner to be the candidate who receives the most popular votes, but rather by a system known as the *Electoral College*, which we will study in Chapter 9. In fact, Trump was the overall winner of the 2016 election even though Clinton received more popular votes than Trump nationwide—65,853,516 for Clinton to 62,984,825 for Trump. This behavior was also exhibited in the 2000 election, where the overall winner, Bush, received only 50,456,002 popular votes nationwide, while Gore received 50,999,897.

election. In other words, Nader was a *spoiler candidate* in the sense that he did not stand a realistic chance of winning the national election, but he may have affected its outcome nonetheless.

Such problems are not inherent to only U.S. elections, of course. A similar situation occurred in the 2002 French presidential election, when in the initial round of voting, political outsider Jean-Marie Le Pen finished second, beating mainstream candidate Lionel Jospin by 0.683% of the votes cast. This prevented an anticipated very close head-to-head final contest between political rivals Jacques Chirac and Jospin. Instead, the final contest was an overwhelming victory for Chirac over Le Pen. Both of these examples serve to illustrate how third-party candidates have the potential to introduce complexities into elections that are not present when only two candidates are involved. In this chapter, we'll begin to investigate these complexities and some ways of dealing with them.

The Plurality Method

As we've seen, Hillary Clinton did not receive a majority of the votes cast in New Hampshire in the 2016 U.S. presidential election. She did, however, receive more votes than any of the other candidates. To state this situation more concisely, we could say that Clinton received a **plurality** of the votes cast in New Hampshire.

Question 2.2.*

- (a) For elections with two candidates, explain why the words *plurality* and *majority* mean exactly the same thing.
- (b) For elections with more than two candidates, explain why the words *plurality* and *majority* do not mean exactly the same thing.

As you saw in Question 2.2, the kind of elections we studied in Chapter 1 (those involving only two candidates) gave us no reason to distinguish between a plurality winner and a majority winner. In fact, we actually used the definition of plurality (without using the word itself) when we defined majority rule in Chapter 1.

Warmup 2.1 demonstrates that the situation is more complicated for elections with more than two candidates. Since it is possible for a candidate to win an election without receiving a majority of the votes cast, we must make a distinction between receiving the *largest* number of votes, and receiving *more than half* of the votes. To do so, we'll adopt the following definitions.

Definition 2.3. Consider an election with more than two candidates.

- **Majority rule** is the voting system that elects the candidate who receives more than half of the votes, if such a candidate exists. If

no candidate receives more than half of the votes, then majority rule results in a tie, with no declared winner.

- The **plurality** method (or just *plurality* for short) is the voting system that elects the candidate who receives the largest number of votes, even if it is less than half of the total number of votes cast. Plurality results in a tie, with multiple winners elected, when two or more candidates receive exactly the same number of votes, and this number is more than the number of votes received by any of the other candidates.

Question 2.4.*

- Which of the two methods described in Definition 2.3 is more likely to result in a tie?
- If a candidate wins an election under majority rule, would that candidate also be guaranteed of winning under plurality?
- If a candidate wins an election under plurality, would that candidate also be guaranteed of winning under majority rule?

Question 2.5. Twenty-one people filed paperwork with the U.S. Federal Election Commission as candidates for the 2016 Republican nomination for president. The total number of votes cast in 2016 Republican primaries nationwide was 31,183,841.

- Donald Trump received 14,015,993 of these votes. Did he receive a majority of the votes cast in the Republican primaries?
- If the winner of the 2016 Republican nomination had been chosen by plurality from these 21 candidates, what is the smallest number of votes Trump could have received and still have had a chance of winning the nomination? (Assume that the number of voters remains fixed at 31,183,841.)
- Under the same assumptions as in part (b), what is the maximum number of voters who could have preferred Trump the *least* among the 21 candidates in order for him to still have had a chance of winning the nomination?
- Using your answers to parts (b) and (c), formulate a well-written criticism of the plurality method. You don't have to agree with your argument, but put yourself in the shoes of a critic and try to predict the type of argument that might be made against plurality.

The Borda Count

Although a number of important elections are decided by the plurality method, it is only one of several voting systems that we'll investigate. For

instance, many collegiate sports polls use a version of a method known as the **Borda count** to form their rankings. The Borda count, which uses a point system to determine overall rankings, is named in honor of Jean-Charles de Borda, a French astronomer, mathematician, military officer, and voting theory pioneer.

Question 2.6.* Part of the 1971 Associated Press preseason college football poll, which was formed using a version of the Borda count, is shown in Table 2.3.

- (a) Which team would have been ranked first if the ranking had been based solely on the number of first-place votes received?
- (b) Did the team from part (a) receive a *majority* of the first place votes?

Rank	Team	Points	First-Place Votes
1	Notre Dame	885	15
2	Nebraska	870	26
3	Texas	662	5
4	Michigan	593	1
5	Southern California	525	1
6	Auburn	434	1
⋮	⋮	⋮	(all 0)
20	Northwestern	58	1
⋮	⋮	⋮	(all 0)

TABLE 2.3. AP college football poll, 1971 preseason

We haven't learned much about the Borda count yet, but we can make one very important observation from Question 2.6: under the Borda count, it is possible for a candidate (or team, as it is in this case) to be viewed as the most desirable by a majority of the voters in an election, but still not win! When a voting system is capable of behaving this badly, we say that the system violates the **majority criterion**. (We'll define this term more precisely in a bit.) Incidentally, it's worth noting that plenty of other anomalies can occur with the Borda count. For instance, in the 1994 Associated Press preseason college football poll, Nebraska received a plurality of the first-place votes (though not a majority), but was ranked *fourth*!

At this point, you might be wondering what kind of strange and perverse system would fail to elect a candidate who is the most desirable choice of more than half of the voters in an election. But would it surprise you to learn that many political scientists and mathematicians have suggested that the Borda count is not strange or perverse at all, but is in fact superior to plurality? We'll investigate this claim more fully later on, but first we need

to take care of a few preliminary details that were not around to bother us when we were considering elections with only two candidates.

Preference Orders

Notice that in a two-candidate election, a voter’s ranking of the candidates from most to least desirable is determined entirely by their first choice. For instance, in the Stickeyville mayoral election from Chapter 1, if I vote for Stutzman, you know that Stutzman is my first choice and Dowell is my second choice. Because there are only two candidates, once you know who I voted for, then you know everything that could possibly be known about my preferences for all of the candidates.

Suppose, however, that I told you I had voted for Hillary Clinton in the 2016 U.S. presidential election. Would you know who my second and third choices were? You might be able to make an educated guess, but without more information, you couldn’t know for sure. In order for you to fully understand my preferences, I would have to provide you with some kind of ranking, or what we’ll call a **preference order** (sometimes also called a *preference ballot* or *preference list*). If we focus only on the three most popular candidates (Hillary Clinton, Donald Trump, and Gary Johnson) and ignore the others, my preference order would have had to have been one of the following:

Rank	Candidate	Rank	Candidate
1	Clinton	1	Clinton
2	Trump	2	Johnson
3	Johnson	3	Trump

To save some ink, I could also specify my preferences in abbreviated form by simply writing $C \succ T \succ J$ for the preference order on the left, and $C \succ J \succ T$ for the one on the right. Note that the \succ symbol is analogous to the “greater than” sign ($>$) that we use to compare numbers; it means “is preferred to” and provides a compact way of listing preferences.

Question 2.7.*

- (a) If you didn’t know that Clinton was my top choice, how many possible rankings could I have had of Clinton, Trump, and Johnson?
- (b) Write down each of the possible rankings from part (a). For each one, provide both a table and a list using \succ notation.
- (c) If I had also included Green Party candidate Jill Stein in my rankings, how many possible rankings could I have had?
- (d) Suppose again that Clinton was my top choice. In how many different ways could I have ranked Clinton, Trump, Johnson, and Stein?

It's important to note that, in many voting systems, only the candidate at the top of each voter's preference order matters. For instance, with plurality or majority rule, the ballot I submit only allows me to vote for my first choice. In my mind, I may not have even formulated my preferences among the other candidates. Perhaps I simply thought to myself, "I want Clinton to win, and so I don't care about any of the other candidates." Or perhaps I was able to form a ranking of Clinton and Trump, but didn't know enough about any of the other candidates to have any real preference between them. In either case, the voting system that was used to make the decision didn't ask me for any of this additional information. It simply asked me to provide my first choice.

That said, even voting systems that use only the voters' top-ranked choices often yield a natural overall ranking of the candidates. For instance, returning to the 2016 U.S. presidential election, plurality in the state of New Hampshire produces the natural ranking $C \succ T \succ J \succ \dots$, since Clinton received more votes than Trump, who received more votes than Johnson, who received more votes than any of the other candidates.

The ranking of the candidates produced by a voting system is called a **societal preference order**; it can be thought of as the ranking of the candidates that, according to the voting system being used, best represents the will of the voters. Of course, the winner of the election is just the first candidate listed in the societal preference order.

Question 2.8.* Suppose Filiz, Gerald, Helen, and Ivan are all running for the coveted office of President of the Cartoon Voice Actors' Association of Bulgaria (CVAAB). The preference orders of each of the 27 members of the association are given in Table 2.4. Such a table is called a **preference schedule**. The column headings indicate the number of voters with each preference order. For instance, the first column indicates that 12 members of the association have the preference order $F \succ G \succ H \succ I$. Note that only four of the many preference orders possible for this election are represented in the table.

Rank	Number of Voters			
	12	7	5	3
1	F	G	H	I
2	G	H	I	H
3	H	I	F	G
4	I	F	G	F

TABLE 2.4. Preference schedule for the CVAAB election

- (a) Under majority rule, what would the outcome of the election be?
- (b) Under plurality, what would the outcome of the election be, and what societal preference order would be produced?

Question 2.9. Do you think the plurality winner of the CVAAB presidential election in Question 2.8 best represents the will of the voters? If so, explain why. Otherwise, give a convincing argument for why you think some other candidate would be better.

Question 2.10. A critic of the plurality method writes the following:

Under plurality, it is possible for the winner of an election to be ranked first by an arbitrarily small percentage of the electorate and last by an arbitrarily large percentage of the electorate.

Write a statement either in support of or in opposition to this view. Use preference schedules to strengthen your argument. (Hint: You might want to look back at your answer to Question 2.5.)

Back to Borda

Recall that we left the Borda count knowing only that it had the ugly ability to violate the seemingly natural majority criterion. Using the language of the previous section, the majority criterion can be defined as follows:

Definition 2.11. A voting system satisfies the **majority criterion** if whenever a candidate is ranked first by a majority of the voters, that candidate will be ranked first in the resulting societal preference order.

Question 2.12.* Do the 2016 U.S. presidential election results in the state of New Hampshire prove that the plurality method violates (i.e., does not satisfy) the majority criterion? Why or why not?

Question 2.13. Do you think that the majority criterion is reasonable? Should the Borda count be discarded because it does not satisfy the majority criterion?

So in what bizarre way does the Borda count work in order to make it capable of violating the majority criterion? The next definition provides the details.

Definition 2.14. Consider an election with n candidates. The **Borda count** works as follows:

- Each voter submits a ballot that contains their entire preference order for all the candidates in the election.
- For each ballot cast, points are awarded to each candidate according to the following rules:
 - A first-place ranking is worth $n - 1$ points.
 - A second-place ranking is worth $n - 2$ points.

- A third-place ranking is worth $n - 3$ points.
- ⋮
- A last-place (n th-place) ranking is worth $n - n = 0$ points.
- The candidate who accumulates the largest number of points from all of the ballots is declared the winner, and the societal preference order is determined by listing the candidates according to the number of points they receive, from largest to smallest. The Borda count produces a tie when two or more candidates receive exactly the same number of points. Candidates who receive identical point totals occupy consecutive indistinguishable positions in the societal preference order. In the event that two or more candidates are tied with the largest number of points, all such candidates are declared winners.

Question 2.15.* Under the Borda count, what would the outcome of the CVAAB presidential election from Question 2.8 be? What societal preference order would be produced?

Question 2.16. Does the definition of the Borda count seem strange or unreasonable to you? If so, explain how. Otherwise, discuss the apparent contradiction in the fact that a reasonably defined system like the Borda count can violate the majority criterion.

Question 2.17. In light of your answers to Questions 2.8, 2.9, and 2.15, who do you think should win the CVAAB presidential election? Give a convincing argument to justify your answer.

May's Theorem Revisited

In this chapter, we looked at two voting systems: plurality and the Borda count. We evaluated the fairness of these systems primarily by means of the majority criterion; we saw that plurality does satisfy the majority criterion, whereas the Borda count does not.

Let's not forget, however, that we also have three criteria from Chapter 1 that we could use to evaluate these systems: anonymity, neutrality, and monotonicity. The definition of anonymity translates naturally to situations involving more than two candidates, while neutrality and monotonicity require only slight modifications.

Definition 2.18.

- A voting system is **anonymous** if it treats all of the voters equally, meaning that if any two voters traded preference orders, the outcome of the election (and the resulting societal preference order) would remain the same.
- A voting system is **neutral** if it treats all of the candidates equally, meaning that if *every* voter switched the positions of two particular

candidates in their individual preference orders, the positions of these two candidates would switch in the resulting societal preference order as well.

- A voting system is **monotone** if changes favorable only to a particular candidate in individual preference orders cannot cause that candidate to finish lower in the resulting societal preference order.

It's important to note that, in Definition 2.18, the phrase *changes favorable only to a particular candidate* means changes that involve nothing more than increasing that candidate's rank on one or more of the individual voters' preference ballots.

Question 2.19. Clearly explain why, in a monotone voting system, changes *unfavorable* only to a particular candidate in any number of individual preference orders cannot cause that candidate to finish *higher* in the resulting societal preference order.

Question 2.20.* Suppose the CVAAB presidential election from Question 2.8 is decided by a voting system that, given the 27 preference orders shown in Table 2.4, would produce the societal preference order $H \succ F \succ G \succ I$.

- (a) If the voting system were neutral, what societal preference order would be produced if the members of the CVAAB changed their preference ballots to those shown in Table 2.5?

	Number of Voters			
Rank	12	7	5	3
1	<i>F</i>	<i>G</i>	<i>I</i>	<i>H</i>
2	<i>G</i>	<i>I</i>	<i>H</i>	<i>I</i>
3	<i>I</i>	<i>H</i>	<i>F</i>	<i>G</i>
4	<i>H</i>	<i>F</i>	<i>G</i>	<i>F</i>

TABLE 2.5. Revised CVAAB preference schedule 1

- (b) Again assuming only neutrality, what can you say about the societal preference order that would be produced if the CVAAB members changed their preference ballots to those shown in Table 2.6?

	Number of Voters			
Rank	12	7	5	3
1	<i>F</i>	<i>G</i>	<i>I</i>	<i>H</i>
2	<i>G</i>	<i>I</i>	<i>H</i>	<i>I</i>
3	<i>I</i>	<i>H</i>	<i>F</i>	<i>F</i>
4	<i>H</i>	<i>F</i>	<i>G</i>	<i>G</i>

TABLE 2.6. Revised CVAAB preference schedule 2

- (c) Assuming the voting system is both neutral *and* monotone, what can you say about the societal preference order that would be produced if the CVAAB members changed their preference orders to those shown in Table 2.6?

Question 2.21.

- (a) Which of the properties of anonymity, neutrality, and monotonicity are satisfied by plurality? Which of these three properties are not satisfied? Give a convincing argument to justify each of your answers.
- (b) Which of the properties of anonymity, neutrality, and monotonicity are satisfied by the Borda count? Which of these three properties are not satisfied? Give a convincing argument to justify each of your answers.
- (c) Do either of your answers to parts (a) or (b) contradict May's Theorem? Explain.

Questions for Further Study

Question 2.22.

- (a) Is plurality a quota system? Give a convincing argument to justify your answer.
- (b) Does your answer to part (a) contradict Theorem 1.22? Explain. (Hint: Consider your answer to Question 2.21.)

Question 2.23. How many first-place votes must a candidate receive in order to win an election if the Borda count is used to determine the winner? Give an example using a preference schedule to justify your answer.

Question 2.24. Write a short biography of Jean-Charles de Borda, including his most important contributions both inside and outside voting theory, and some information about his military career.

Question 2.25. Suppose that the Stickeyville Board of Commissioners is proposing that their next mayoral election be decided by the Borda count instead of plurality. Based on what you've learned in this chapter, write a formal letter to the editor of the Stickeyville Daily Review either in support of or opposition to this proposal. Regardless of your personal feelings on the issue, your letter should discuss the pros and cons of each system and specifically address the fact that the Borda count does not satisfy the majority criterion.

Question 2.26. In the CVAAB presidential election from Question 2.8, suppose that the 3 voters represented in the far right column of Table 2.4 changed their preference orders from $I \succ H \succ G \succ F$ to $I \succ H \succ F \succ G$.

- (a) Did any of the voters change their preference between just Gerald and Helen?
- (b) Under the Borda count, how would the societal preference order change as a result of these individual changes?
- (c) In light of your answer to Question 2.15, does anything seem strange about your answers to parts (a) and (b) above? Explain.
- (d) Without moving Filiz above Gerald or Helen in any individual preference order, find a way to modify the preference schedule in Table 2.4 so that Filiz wins the election under the Borda count.

Question 2.27. Suppose a small group of voters in the CVAAB presidential election from Question 2.8 are strongly opposed to Filiz being elected as their new president. Explain how these voters could manipulate the plurality method to reduce Filiz's chances of winning. (Hint: Suppose the voters introduced and supported another candidate. Whose views should this new candidate emulate in order to hurt Filiz's candidacy the most?)

Question 2.28.

- (a) Find a U.S. presidential election in which the winning candidate received a plurality but not a majority of the nationwide popular vote.
- (b) Find a U.S. presidential election other than the 2016 and 2000 elections in which the winning candidate did not receive a plurality of the nationwide popular vote.

Question 2.29. If the Borda count had been used instead of plurality to determine the winner, who do you think would have won the 2016 U.S. presidential election in New Hampshire? Give a convincing argument to justify your answer. (Hint: You will have to make some conjectures about the preferences of those who voted for Johnson. You may need to do some research to see which of the other major candidates' views most closely resembled Johnson's.)

Question 2.30.

- (a) If the Borda count had been used instead of plurality to determine the winner in Florida, who do you think would have won the 2000 U.S. presidential election? Give a convincing argument to justify your answer. (Hint: You will have to make some conjectures about the preferences of those who voted for Nader. You may need to do some research to see which of the other major candidates' views most closely resembled Nader's.)
- (b) On May 19, 2004, roughly six months before the 2004 U.S. presidential election, Democratic presidential candidate John Kerry met privately with Ralph Nader, who, months earlier, had announced his

candidacy in the election. Why do you think Kerry arranged this meeting? What do you think the two candidates discussed?

- (c) Why do you think the Republican Leadership Council ran pro-Nader ads in the weeks prior to the 2000 U.S. presidential election?

Question 2.31. Some political scientists have speculated that if Vermont senator Bernie Sanders (an independent who caucuses with the Democrats) had been a candidate in the 2016 U.S. presidential election as an independent, he would have won a head-to-head contest against either Hillary Clinton or Donald Trump.

- (a) Had Sanders been a candidate, who do you think would have won the election? Do you think the race would have been close or not? Explain.
- (b) Would your answer to part (a) be different if the Borda count were used instead of plurality to determine the winner of the election in each state? Explain.

Question 2.32. On October 7, 2003, residents of the state of California voted to recall then-governor Gray Davis and replace him with Hollywood actor and former Mr. Olympia Arnold Schwarzenegger. Investigate this election, and write a detailed summary of your findings. Include in your summary answers to at least the following questions:

- How many candidates were listed on the ballot as potential replacements for Davis in case he was recalled?
- How many votes were cast statewide? Did Schwarzenegger receive a majority of them?
- Given the number of candidates and number of votes cast statewide, and the fact that the winner was chosen using plurality, what is the smallest number of votes Schwarzenegger could have received and still have had a chance of winning the election? What is the maximum number of voters who could have preferred Schwarzenegger the *least* among the potential replacement candidates in order for him to still have had a chance of winning the election?
- What legal requirement was met by the citizens of California in order for the recall election to take place?
- What were the main arguments in favor of Davis being recalled?
- Who were some of the more famous candidates to replace Davis? What were their general views?
- How did the actual voting take place?
- What questions were on the ballot?
- What were the results for each question on the ballot?

Question 2.33. The top three teams from the Associated Press college football poll that was released on December 5, 2004 are shown in Table 2.7. The poll was formed using a version of the Borda count and was based on the votes of 65 voters, each of whom ranked 25 teams. The only difference between the version of the Borda count used to form this poll and the Borda count as it was presented in this chapter is that, instead of awarding points for each ballot from 24 for a first-place ranking down to 0 for a last-place ranking (as described in Definition 2.14), the poll awards points for each ballot from 25 for a first-place ranking down to 1 for a last-place ranking. (This is to account for the fact that there are more than 25 teams that could be ranked, and the team with the last-place ranking on each ballot needs to be distinguished from the teams left off the ballot.) Based on this information and the information given in the Table 2.7, is it possible to conclude that all 65 voters had Southern California, Oklahoma, and Auburn ranked in some order as their top three choices? Why or why not?

Rank	Team	Points	First-Place Votes
1	Southern California	1599	44
2	Oklahoma	1556	14
3	Auburn	1525	7
\vdots	\vdots	\vdots	(all 0)

TABLE 2.7. AP college football poll, December 5, 2004

Question 2.34. The top three teams from the Coaches' college football poll that was released on October 12, 2014 are shown in Table 2.8. This poll was formed using the same version of the Borda count as the poll described in Question 2.33, and was based on the votes of 62 voters. Does this poll illustrate a violation of the majority criterion? If so, explain why. Otherwise, state the number of additional first-place votes Florida State would have needed to receive in order for this ranking to illustrate such a violation.

Rank	Team	Points	First-Place Votes
1	Mississippi State	1490	26
2	Florida State	1489	31
3	Mississippi	1436	5
\vdots	\vdots	\vdots	(all 0)

TABLE 2.8. Coaches' college football poll, October 12, 2014

Question 2.35. Consider again the Coaches' college football poll that was released on October 12, 2014, for which the top three teams are shown in Table 2.8. The reason this was called the *Coaches'* poll is because the voters in the poll were a collection of actual college football coaches, including possibly the coaches at Mississippi State and Florida State. Explain how the coaches at Mississippi State and Florida State, were they in fact voters, could have easily manipulated the results of the poll.

Question 2.36. Each year, the Most Valuable Player in major league baseball's National League is chosen by a group of 30 sportswriters using a variation of the Borda count, which differs from the Borda count as it was presented in this chapter only in the number of points assigned to the players on each ballot. The winners of the 2015 and 2016 National League MVP awards and the number of points/votes they received are shown in Table 2.9. From the information in the table, determine the number of points awarded for first and second-place votes on National League MVP ballots.

Year	Winner	Points	1st-Place Votes	2nd-Place Votes
2015	Bryce Harper	420	30	0
2016	Kris Bryant	415	29	1

TABLE 2.9. National League MVP winners, 2015 and 2016

Question 2.37. As noted in Question 2.36, major league baseball's Most Valuable Players are chosen at the end of each season by a group of sportswriters using a variation of the Borda count. Investigate the result of the American League MVP voting after the 2001 baseball season, and determine if the result would have been the same if the Borda count had been used exactly as it was presented in this chapter.

Question 2.38. Find out how voting is conducted for the Heisman Memorial Trophy, and write a detailed summary of your findings. Include in your summary at least a description of what the Heisman Trophy is given for, who votes, how the voting is conducted, and the exact voting system used to declare the winner. In addition, use the 2008 vote results to illustrate the calculations required for this voting system, and comment on anything about these results that seems surprising or unusual to you.

Question 2.39. Find a magazine, newspaper, or web site that describes an example not related to sports where the Borda count or a version of the Borda count was used to arrive at some type of decision or ranking. Write a summary of your findings, including the name of your source, the outcome of the example, and how the Borda count was used.

Question 2.40. Decide whether each of the following statements are true or false. Give a convincing argument or example to justify each of your answers.

- (a) In a three-candidate election that does not result in a tie, the Borda count winner must receive at least one first-place vote.
- (b) In a four-candidate election that does not result in a tie, the Borda count winner must receive at least one first-place vote.

Question 2.41. Find a copy of the article “Would the Borda Count Have Avoided the Civil War?” by Alexander Tabarrok and Lee Spector in the *Journal of Theoretical Politics*. Write a summary of the article, including the authors’ answer to the question posed in the article’s title.

Question 2.42. Consider an election with four candidates and the preferences shown below:

	Number of Voters		
Rank	51	25	24
1	<i>A</i>	<i>C</i>	<i>D</i>
2	<i>B</i>	<i>B</i>	<i>B</i>
3	<i>C</i>	<i>D</i>	<i>C</i>
4	<i>D</i>	<i>A</i>	<i>A</i>

- (a) Who would win this election under any system that satisfies the majority criterion?
- (b) Who would win this election under the Borda count?
- (c) Which of the outcomes from parts (a) and (b) do you think is most fair? In your opinion, which best represents the will of the voters?
- (d) Do your answers to parts (a)–(c) affect your opinion of the majority criterion in any way? Explain.

Answers to Starred Questions

- 2.2.** (a) In an election with only two candidates, one candidate cannot receive more votes than the other candidate without receiving *more than half* of the votes. Likewise, one candidate cannot receive more than half of the votes without receiving more votes than the other candidate. Otherwise, the number of votes would exceed the number of voters.
- (b) In an election with more than two candidates, one candidate could receive more votes than any of the others, but still not receive *more than half* of the votes cast. The 2016 U.S. presidential election in the state of New Hampshire is an example of this.

- 2.4.** (a) Majority rule results in a tie whenever none of the candidates in an election receive more than half of the votes cast. For plurality to result in a tie, two or more of the candidates would have to receive the *exact same* number of votes *and* more votes than all of the other candidates. Thus, majority rule is much more likely to result in a tie.
- (b) For a candidate to win an election under majority rule, they must have received more than half of the votes cast. Since none of the other candidates could have also received more than half of the votes, the candidate who did receive more than half would have necessarily received more votes than any of the other candidates, and thus be guaranteed of also winning under plurality.
- (c) A plurality winner of an election need not also win under majority rule. For example, although Hillary Clinton was the plurality winner of the 2016 U.S. presidential election in New Hampshire, she would not have won under majority rule since she did not receive a majority of the votes cast.

2.6. Nebraska would have been ranked first with a majority (26 out of 50) of the first-place votes.

- 2.7.** (a) Since there are 3 choices for my first-place candidate, 2 choices for my second-place candidate (one of the two that I didn't rank first), and only 1 choice for my third-place candidate (the one that I didn't rank either first or second), it follows that there are $3 \times 2 \times 1 = 6$ different ways in which I could have ranked the candidates.
- (b) The six possible rankings are the two displayed in the text, along with the four shown in the following table.

Rank	Candidate	Candidate	Candidate	Candidate
1	Trump	Trump	Johnson	Johnson
2	Clinton	Johnson	Clinton	Trump
3	Johnson	Clinton	Trump	Clinton

- (c) Similar reasoning as in part (a) applies. With four candidates, there are $4 \times 3 \times 2 \times 1 = 24$ different ways in which I could have ranked the candidates.
- (d) If Clinton must be ranked first, then I would need to rank the remaining three candidates. There are $3 \times 2 \times 1 = 6$ different ways to do so.
- 2.8.** (a) Under majority rule, the election would result in a tie (with no winner elected), since no candidate would receive more than half of the first-place votes cast.

- (b) Under plurality, Filiz would win, since she would receive more first-place votes (12) than any of the other three candidates. The societal preference order would be $F \succ G \succ H \succ I$.

2.12. The 2016 U.S. presidential election results in the state of New Hampshire *do not* prove that the plurality method violates the majority criterion. The majority criterion states that if a candidate in an election receives more than half of the votes cast, then that candidate should be declared the winner. It does not, however, state that if a candidate receives less than half of the votes, then that candidate should not be declared the winner. The plurality method actually satisfies the majority criterion. As we argued in Question 2.4, a majority rule winner would always be declared a winner under plurality as well.

2.15. According to the preference schedule in Question 2.8, Filiz would receive 12 first-place votes, 0 second-place votes, 5 third-place votes, and 10 last-place votes. Since there are 4 candidates in the election, first-place votes are worth 3 points each, second-place votes are worth 2 points, third-place votes 1 point, and last-place votes 0 points. Thus, Filiz would be awarded $(12 \times 3) + (0 \times 2) + (5 \times 1) + (10 \times 0) = 41$ points. Similar calculations show that Gerald would be awarded 48 points, Helen 47 points, and Ivan 26 points. Thus, the winner under the Borda count would be Gerald, and the resulting societal preference order would be $G \succ H \succ F \succ I$.

- 2.20.** (a) Since all of the voters swapped the positions of H and I in their individual preference orders, neutrality dictates that H and I would have to be swapped in the resulting societal preference order, yielding $I \succ F \succ G \succ H$.
- (b) Although all of the voters swapped the positions of H and I in their preference orders, the three voters represented in the rightmost column of the table also swapped F and G . Since neutrality only dictates changes to the societal preference order when each voter swaps exactly the same candidates, we have no way of knowing what the new societal preference order would be in this case.
- (c) Notice first that the preference schedule in Table 2.6 differs from the one in Table 2.5 only by a swap of F and G in the 3 voters' preference orders represented in the rightmost column of the tables, a change that is favorable only to F . By part (a), we know that the preference schedule in Table 2.5 produces the societal preference order $I \succ F \succ G \succ H$. Since the preference schedule in Table 2.6 can be obtained from the one in Table 2.5 by changes favorable only to F (and unfavorable to G), monotonicity requires that F remain preferred to G and H in the resulting societal preference order. However, there are many societal preference orders

that would be consistent with these requirements. To list a few:

$$I \succ F \succ G \succ H,$$

$$F \succ I \succ G \succ H,$$

and even the somewhat strange

$$F \succ I \succ H \succ G.$$

(Note the reversal of the ranking of G and H in this last order.)