Chapter 3

Back into the Ring

Focus Questions

In this chapter, we'll explore the following questions:

- What is the Condorcet winner criterion? Which voting systems satisfy the Condorcet winner criterion, and which do not?
- What is sequential pairwise voting, and how does it work?
- What is instant runoff voting? How does it work, and where is it commonly used?
- What are some of the advantages and disadvantages of sequential pairwise voting and instant runoff?

Warmup 3.1. Suppose Skip, Norm, and Jesse are all running for President of the 10,000 Lakes Club, with the preferences of the 100 members of the club as shown in Table 3.1.

	Number of Voters			
Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

TABLE 3.1. Preference schedule for the 10,000 Lakes Club

(a) What would be the outcome of the election under majority rule?

(b) What would be the outcome of the election under plurality?

(c) What would be the outcome of the election under the Borda count?

(d) Which candidate is ranked first by the largest number of voters?

(e) Which candidate is ranked last by the largest number of voters?

(f) In a head-to-head contest[1] between just Skip and Norm, who would win?

(g) In a head-to-head contest between just Skip and Jesse, who would win?

(h) In a head-to-head contest between just Norm and Jesse, who would win?

(i) Does anything about your answers to parts (a)–(h) above strike you as being strange or unusual? Explain.

As you probably noticed, the election from Warmup 3.1 exhibits a number of peculiarities. For one thing, the plurality winner, Jesse, is ranked first by only 37% of the voters. The other 63% rank him last; they would all prefer either Skip or Norm. Furthermore, Jesse would lose to either of the other two candidates if he went up against them head-to-head.

It may be tempting to dismiss these observations as features of a contrived example that would never occur in real life... unless you're from Minnesota or are familiar with the state's 38th governor, former professional wrestler and radio shock-jock Jesse "The Body" Ventura. In 1998, Ventura, running as a Reform Party candidate, claimed a stunning victory over Minnesota Attorney General Skip Humphrey (a Democrat) and St. Paul Mayor Norm Coleman (a Republican) in the state's gubernatorial race. Ventura won under plurality by receiving 37% of the popular votes, higher than the percentages received by either Humphrey (28%) or Coleman (35%). Although it is impossible to know exactly how Minnesota's voters ranked Ventura in comparison to the other candidates, many have speculated that the voters' preferences looked a lot like those in Table 3.1, with a large number of voters ranking Ventura in last place.[2]

For a more recent example, consider the 2016 Republican primaries in the U.S. presidential election. One public opinion poll, conducted in March of 2016, suggests that Donald Trump would have won a plurality election against Ted Cruz, John Kasich, and Marco Rubio, but would have lost to each of these candidates—by double-digit margins—if he went up against them head-to-head [**31**].

Outside the U.S., the plurality winner in the initial round of voting in the 2017 French presidential election received less than 25% of the votes

[1] By *head-to-head contest*, we mean a two-candidate election with the winner decided by majority rule. Thus, when we say that A would defeat B in a head-to-head contest, we mean that if the voters were forced to choose between only these two candidates, then A would receive more votes than B.

[2] For a well-researched opposing view, see [**32**].

cast, with the second, third, and fourth-place candidates all trailing by less than 5%. However, in spite of the close outcome in this initial round, the winner, Emmanuel Macron, handily defeated his opponent, Marine Le Pen, in the second round, winning by a 2-to-1 (66% to 34%) margin.

Now back to Minnesota: In the weeks and months following the 1998 gubernatorial election, many political commentators tried to explain how Ventura, whose only prior political experience had been a four-year stint as mayor of Brooklyn Park, Minnesota, was able to defeat two well-known opponents, each having significantly more experience in the political arena. Many suggested (correctly) that Ventura had mobilized more young voters than either of the other two candidates. Others speculated that Ventura's celebrity and larger-than-life personality had earned him the votes of those who were not familiar with the political views of any of the three candidates. Only a handful, however, suggested that Ventura's victory might have been merely a consequence of the voting system that was used: plurality.

This, of course, is what we are most interested in. In Warmup 3.1, we saw how an alternative to plurality—the Borda count—might have produced an outcome that represented the will of Minnesota's voters better than the plurality outcome. But, as we saw in the last chapter, the Borda count is not without flaws; for example, it is capable of violating the majority criterion, a desirable property that even plurality satisfies.

As we're beginning to see, deciding elections with more than two candidates can be tricky. There are a lot of thorny issues to deal with, and we'll have to wrestle with these issues for a while (pun definitely intended) if we are to have any hope of resolving them. That's exactly what we'll do in this chapter.

Condorcet Winners and Losers

In Warmup 3.1, we made a couple of important but troublesome observations. First, we saw that plurality can fail to elect a candidate who would win a head-to-head contest against each of the other candidates. Second, and even more disturbing, we saw that plurality can elect a candidate who would *lose* a head-to-head contest against each of the other candidates.

These kinds of phenomena are often attributed to the well-named Marie Jean Antoine Nicolas de Caritat, the Marquis de Condorcet, a French mathematician and contemporary of Borda, who is usually referred to simply by the name *Condorcet*. The following terms, which correspond to the ideas described above, were named in honor of Condorcet.

Definition 3.2.

- A **Condorcet winner** is a candidate in an election who would win a head-to-head contest (with the winner decided by majority rule) against each of the other candidates.

- A **Condorcet loser** is a candidate in an election who would lose a head-to-head contest (with the winner decided by majority rule) against each of the other candidates.
- A voting system that will always elect a Condorcet winner, whenever one exists, is said to satisfy the **Condorcet winner criterion** (**CWC** for short).
- A voting system that will never elect a Condorcet loser is said to satisfy the **Condorcet loser criterion** (**CLC** for short).

Question 3.3.* Assuming the data in Table 3.1 accurately reflects the preferences of the voters in the 1998 Minnesota gubernatorial election, was there a Condorcet winner and/or loser in the election? If so, who?

Knowing that a voting system satisfies the Condorcet loser criterion is useful; after all, such a system would avoid the possibility of electing a candidate who would consistently lose in head-to-head contests. But why should we stop there? After all, the Condorcet winner criterion seems entirely reasonable and is perhaps even more important because it declares a condition that has the potential to yield a winner, whereas the Condorcet loser criterion simply rules out certain candidates. And if there is a candidate in an election who would win a head-to-head contest against any of their opponents, why shouldn't that candidate be elected? With this in mind, our goal for the next few pages will be to find a voting system that satisfies the CWC—and hopefully the other desirable properties we've considered as well. Before we do so, however, let's pause to consider a few other important details.

Question 3.4.* Consider the preference schedule in Table 3.2.

	Number of Voters		
Rank	1	1	1
1	A	B	C
2	B	C	A
3	C	A	B

TABLE 3.2. Condorcet's preference schedule

(a) In a head-to-head contest between just candidates A and B, who would win?

(b) In a head-to-head contest between just B and C, who would win?

(c) In a head-to-head contest between just A and C, who would win?

(d) Does anything about your answers to parts (a)–(c) strike you as being strange or unusual? Explain.

(e) Is there a Condorcet winner and/or loser in this election? Explain.

Question 3.4 shows that that it is possible for an election to have neither a Condorcet winner nor a Condorcet loser. This explains why the phrase "when one exists" is a necessary part of the definition of the Condorcet winner criterion. But let's suppose for a minute that we are considering an election in which there is a Condorcet winner. Can there be more than one?

Question 3.5.* Assume that, in a certain election, there are two different Condorcet winners. Explain why this assumption leads to a contradiction to the definition of a Condorcet winner, and why this contradiction shows that it is impossible for an election to have more than one Condorcet winner.

Question 3.6. Use the same kind of reasoning as in Question 3.5 to explain why it is impossible for an election to have more than one Condorcet loser.

We're now ready to look for a voting system that satisfies the Condorcet winner criterion. Let's begin with the systems we investigated in Chapters 1 and 2. As we saw in Warmup 3.1, plurality is not a good option if we want to elect Condorcet winners and avoid electing Condorcet losers. But what about majority rule or the Borda count?

Question 3.7.*

(a) Explain why, whenever majority rule does not result in a tie, the majority rule winner will be a Condorcet winner.

(b) Does your answer to part (a) imply that majority rule satisfies the Condorcet winner criterion? If so, explain why. Otherwise, give an example to show that majority rule can violate the CWC.

(c) Does your answer to part (a) imply that majority rule satisfies the Condorcet loser criterion? If so, explain why. Otherwise, give an example to show that majority rule can violate the CLC.

(d) Are there any special types of elections for which majority rule does satisfy the CWC? Give a convincing argument to justify your answer.

(e) Use your answer to part (a) to explain why any voting system that violates the majority criterion must also violate the CWC.

(f) Use your answer to part (e) to explain why the Borda count violates the CWC.

So it looks like the well has run dry. We are going to have to invent or discover some new voting system if we are to have a chance of satisfying the elusive Condorcet winner criterion. But first, let's take a moment to formalize the observation we made in part (e) of Question 3.7. What we said there was that any voting system that violates the majority criterion

must also violate the CWC. Likewise, we could also say that any voting system that satisfies the CWC must also satisfy the majority criterion. In other words, the Condorcet winner criterion is a stronger condition than the majority criterion. For reference, we'll state these observations in the following theorem.

Theorem 3.8.

- *If a candidate in an election receives a majority of the first-place votes, then that candidate will be a Condorcet winner.*
- *If a voting system satisfies the Condorcet winner criterion, then it will also satisfy the majority criterion.*
- *If a voting system violates the majority criterion, then it will also violate the Condorcet winner criterion.*

Now let's see if we can find a new voting system that satisfies the Condorcet winner criterion.

Sequential Pairwise Voting

Since the definition of a Condorcet winner involves head-to-head, two-candidate elections, a good bet for a system that satisfies the Condorcet winner criterion would be one that uses head-to-head contests to determine the winner. Since we are considering elections with more than two candidates, we'll obviously need to hold more than just a single two-candidate election. But perhaps if we conducted a *sequence* of two-candidate elections, we would be able to collect enough information to determine a winner. If we were lucky, this winner would be the Condorcet winner for the election, just as we wanted.

To illustrate one such method, let's return to the CVAAB presidential election from Question 2.8. Recall that the preferences of the 27 members of the CVAAB are as shown in Table 3.3.

	Number of Voters			
Rank	12	7	5	3
1	F	G	H	I
2	G	H	I	H
3	H	I	F	G
4	I	F	G	F

TABLE 3.3. Preference schedule for the CVAAB election

Remember also that, for this election, the plurality method produced the societal preference order $F \succ G \succ H \succ I$, whereas the Borda count produced $G \succ H \succ F \succ I$. Let's now see what societal preference order

would be produced by a sequence of head-to-head contests. Here's how we'll run the election:

Step 1: First, we'll ask the voters to choose between just Gerald and Helen. Since this is a two-candidate election, we'll use majority rule to decide the winner.

Step 2: Next, we'll ask the voters to choose between Filiz and the winner from Step 1, again using majority rule to decide the winner.

Step 3: Finally, we'll ask the voters to choose between Ivan and the winner from Step 2. Whoever wins this third head-to-head contest will be declared the overall winner of the election.

Question 3.9.*

(a) Under the method described in Steps 1–3 above, who would win the CVAAB presidential election?

(b) Under the method described in Steps 1–3 above, what societal preference order do you think would best represent the will of the voters in the election?

(c) In light of the plurality and Borda count results for this election, does anything about your answers to parts (a) and (b) strike you as being strange or unusual? Explain.

(d) Is there a Condorcet winner and/or loser in this election? Explain.

While your answer to Question 3.9 may not shed a lot of light on who really should be elected as the next CVAAB president, it does at least illustrate a way to use head-to-head contests to determine the winner of an election with more than two candidates. This voting system is known as **sequential pairwise voting**.

Before going on, we need to stop for a moment and consider one very important difference between sequential pairwise voting and the other systems we've already considered. Notice that with plurality and the Borda count, we had no problems constructing societal preference orders based on the results of the election. Because of this, you may have assumed that you could do the same thing in part (b) of Question 3.9. In fact, you might have assembled the results of each head-to-head contest to arrive at the societal preference order $I \succ F \succ G \succ H$. This ordering seems quite natural and is consistent with the results of each head-to-head contest we conducted. But what about the pairs of candidates that didn't compete directly against each other—for example, Ivan and Gerald? It would seem natural to conclude that if Ivan beats Filiz, and Filiz beats Gerald, then Ivan should certainly beat Gerald. As it turns out, however, this is not always the case; in this example, Ivan would actually *lose* to Gerald (by a whopping margin!) if the voters were asked to decide between just the two of them. Look back to

Question 3.4 to see another example of this type of unexpected behavior. Something similar happened there, and you may have even pointed it out in your answer to part (d) of that question.

What you've probably noticed by now is that voting theory is full of mind-boggling anomalies, which we often call voting **paradoxes**.[3] The example in Question 3.4 is called **Condorcet's paradox** and is quite famous in the world of election perversities.

Returning to our discussion of societal preference orders, the point that must be made is that sequential pairwise voting does not always produce a well-defined societal preference order. There are several ways in which we could deal with this, but for now we'll handle it by simply defining societal preference orders under sequential pairwise voting so that the winning candidate is ranked first, and all of the other candidates are tied for second (or last, depending on how you look at it). In the example from Question 3.9, we would represent this order by $I \succ F \approx G \approx H$. Notice that, in the same way we used a fancy version of the greater than symbol to indicate a preference between two candidates ($\succ$ instead of $>$), we'll use a fancy version of the equals symbol to indicate a tie between two candidates ($\approx$ instead of $=$). And from now on we'll use this notation in cases where plurality or the Borda count or any other voting system results in a tie between candidates in a societal preference order.

And now one last detail: Those of you who are really on your toes might be somewhat skeptical of the solution we just proposed. After all, even the preference order $I \succ F \approx G \approx H$ suggests that I would beat G in a head-to-head contest, which we know not to be the case. If you're thinking this, you're absolutely right—we haven't completely solved the problem that we set out to solve. But we have made the situation somewhat better by eliminating the potentially false comparisons between losing candidates. Also, remember that, even though G would beat I in a head-to-head contest, I is the unique winner under sequential pairwise voting. In that sense, I really is preferred to G, just as Ventura was preferred to both Coleman and Humphrey in the 1998 Minnesota gubernatorial election (as decided by plurality), even though either of these opponents may have been able to defeat him in a head-to-head contest. We can see then that, as is the case with many voting systems, there is more to sequential pairwise voting than what first meets the eye. As the late radio commentator Paul Harvey used to say, in a few moments you'll know "the rest of the story." But first, let's see how sequential pairwise voting stacks up with regard to Condorcet's properties.

[3] Webster's Dictionary defines a paradox to be a tenet or proposition contrary to received opinion; an assertion or sentiment seemingly contradictory, or opposed to common sense; that which in appearance or terms is absurd, but yet may be true in fact.

Question 3.10.*

(a) Could a Condorcet winner ever lose a head-to-head contest with another candidate? Why or why not?

(b) What does your answer to part (a) allow you to conclude about sequential pairwise voting and the Condorcet winner criterion?

Question 3.11. Does sequential pairwise voting satisfy the Condorcet loser criterion? If so, explain why. Otherwise, give an example of a preference schedule for which sequential pairwise voting would elect a Condorcet loser.

So, apart from the problem in defining societal preference orders, is sequential pairwise voting a good system for choosing the winner of an election with more than two candidates? In at least one regard, it looks quite promising. After all, sequential pairwise voting will *always* elect a Condorcet winner when one exists. But what happens in situations when there is no Condorcet winner, as was the case in the CVAAB presidential election?

To answer this question, first observe that in sequential pairwise voting, we must specify—before the election takes place—the order in which the candidates will compete against each other. This seemingly harmless sequence is called the **agenda** and is usually denoted by simply listing the candidates in the order in which they are to be introduced into the comparisons. For example, we used the agenda G, H, F, I in the CVAAB presidential election from Question 3.9.

Question 3.12.* Who would win the CVAAB presidential election using sequential pairwise voting with the agenda F, G, H, I?

Question 3.13.

(a) Find a sequential pairwise voting agenda for which Filiz would win the CVAAB presidential election.

(b) Find a sequential pairwise voting agenda for which Gerald would win the CVAAB presidential election.

By now, you should be convinced that sequential pairwise voting, although avoiding some pitfalls, has at least one significant problem of its own. Questions 3.12 and 3.13 illustrate how, in the absence of a Condorcet winner, the agenda can play an inordinately powerful role in determining the winner of the election.[4] This property indicates that sequential pairwise voting is highly manipulable, a fact that has not gone unnoticed by politicians

[4]Incidentally, there is an obvious similarity between the lose-once-and-you're-out philosophies of sequential pairwise voting and single elimination tournaments in which a participant can be declared the winner without having to win head-to-head contests against all or even most of the other participants. The fact that sequential pairwise voting is highly dependent on the agenda chosen is illustrated by the fact that in many single elimination tournaments, the participants expected to perform the best are often "seeded" in such a way that they would meet each other as late as possible in the tournament.

and other decision-makers who rely on it. Because the agenda can give an advantage to specific candidates—while putting others at a disadvantage—it is reasonable to suspect that sequential pairwise voting may also violate the fundamental property of neutrality. Unfortunately, our next question confirms this suspicion.

Question 3.14. Suppose that all of the voters in the CVAAB presidential election switched the positions of I and H in their preference orders, yielding the new preference schedule shown in Table 3.4.

	Number of Voters			
Rank	12	7	5	3
1	F	G	I	H
2	G	I	H	I
3	I	H	F	G
4	H	F	G	F

TABLE 3.4. Revised CVAAB preference schedule

(a) Using sequential pairwise voting with the agenda G, H, F, I, what societal preference order would result from this new preference schedule?

(b) Explain why your answers to Question 3.9 and part (a) of this question show that sequential pairwise voting is not neutral.

So what should we do? Should we give up on sequential pairwise voting, or does it perhaps have some redeeming qualities that make it worth our consideration? Since sequential pairwise voting does arise naturally in a number of important situations, one would assume that it has some desirable features that compensate for its flaws. The Condorcet winner criterion is one such feature—and in elections with a Condorcet winner, the neutrality issue is moot, since the Condorcet winner will be selected regardless of the agenda. Moreover, it turns out that sequential pairwise voting is both anonymous and monotone.

Question 3.15. Explain why sequential pairwise voting is both anonymous and monotone.

Finally, Theorem 3.8 tells us that sequential pairwise voting also satisfies the majority criterion. And since each voting system we've considered for elections with more than two candidates violates at least one of the desirable properties we've investigated, one could argue that sequential pairwise voting is just as good as any of the others. Of course, it could also be the case that we just haven't let the cat out of the bag yet. Perhaps we just need to look a little bit further to find a voting system that will put an end to this whole discussion.

Instant Runoff

In the mid-1800s, an English lawyer and political reformist named Thomas Hare proposed a voting system that involved successively eliminating candidates until only one, the winner, remained. This system, now known as **single transferable vote**, or **instant runoff**, was well-received at the time and has grown in popularity since. Instant runoff is currently used in some important national elections worldwide—for example, to elect the presidents of India and Ireland, and members of the Australian House of Representatives. Instant runoff is also used to elect the mayors of several large cities, including London, San Francisco, and Minneapolis. According to FairVote, a nonpartisan organization that advocates for electoral reform, "literally hundreds of jurisdictions, organizations and corporations use instant runoff voting to elect leaders."

Does that sound convincing? Well, there's more. In 1860, philosopher John Stuart Mill (who we should admit was a close friend of Hare's) wrote the following in his book *Considerations on Representative Government*:

> *The more [the details of the Hare system] are studied the stronger, I venture to predict, will be the impression of the perfect feasibility of the scheme, and its transcendent advantages. Such and so numerous are these, that, in my conviction, they place Mr. Hare's plan among the very greatest improvements yet made in the theory and practice of government.*

It would be hard to find a much stronger endorsement than this, so let's investigate instant runoff and see if it really is as good as John Stuart Mill and the folks at FairVote would have us believe.

Definition 3.16. The **instant runoff** voting system works according to the following three steps:

Step 1: Each voter in the election submits their entire preference order.

Step 2: The candidate with the fewest first-place votes (or candidates in the case of a tie) is eliminated from each voter's preference order, and the remaining candidates are moved up on each preference order, yielding a new collection of preferences.

Step 3: Step 2 is repeated until only a single candidate remains. This candidate is then declared the winner of the election.

If desired, a societal preference order for the election can be formed by listing the candidates in the reverse of the order in which they were eliminated—that is, by listing the candidates starting with the winner, followed by the last candidate eliminated, and continuing down to the first candidate eliminated.

Question 3.17.* Suppose Amaya, Brandon, Carlos, and Delilah are all running for the position of Chair of the Mathematics Department at Podunk University. The preferences of the 21 members of the department are shown in Table 3.5.

	Number of Voters			
Rank	7	6	5	3
1	A	B	C	D
2	B	A	B	C
3	C	C	A	B
4	D	D	D	A

TABLE 3.5. Preference schedule for PU math chair election

(a) Which candidate would be eliminated first under the instant runoff method? Which would be eliminated second? Third?

(b) Who would win the election under the instant runoff method? What would be the resulting societal preference order?

Now let's check the properties. We'll start with anonymity and neutrality. As with some of the systems we've considered already, it might seem fairly obvious to you that instant runoff is both anonymous and neutral. After all, nothing in its definition mentions anything about individual voters or candidates. This is a contrast to sequential pairwise voting, which uses an agenda that specifically lists each candidate, and dictatorships, which explicitly designate one of the voters as being more important than all of the others. Nevertheless, we've seen our intuition fail us before, so let's refer back to the definitions of anonymity and neutrality to explain why instant runoff satisfies both of these properties.

Question 3.18. Use Definition 2.18 to write a detailed explanation of why instant runoff is both anonymous and neutral.

The definition of monotonicity is slightly more complicated, so we'll consider an example before formulating a general argument.

Question 3.19.* Consider again the election for the PU Mathematics Department Chair from Question 3.17. Suppose that, after a heated debate, the 3 voters from the rightmost column of Table 3.5 change their preferences to $A \succ D \succ C \succ B$. Note that this is a change that is favorable only to Amaya.

(a) With these new preferences, who would win the election under the instant runoff method?

(b) Compare your answer to part (a) of this question with your answer to part (b) of Question 3.17. What conclusions can you make?

Hold on a second! Did you catch what just happened in Question 3.19? How is it that instant runoff, which was supposed to save the day, doesn't even satisfy monotonicity? What's going on here? We'll have to wait to answer that question until the next chapter, but first let's see if there is anything positive we can salvage from Mr. Hare's "perfectly feasible" system.

Question 3.20.

(a) Explain why if, at any stage in the process of instant runoff voting, one candidate receives a majority of the first-place votes, then that candidate can automatically be declared the winner of the election.

(b) Use your answer to part (a) to explain why instant runoff satisfies the majority criterion.

Question 3.21. Consider an election between three candidates with the preference schedule shown in Table 3.6.

	Number of Voters		
Rank	1	2	2
1	A	B	C
2	B	A	A
3	C	C	B

TABLE 3.6. Instant runoff and the CWC

(a) Is there a Condorcet winner for this election?

(b) Who would win the election under the instant runoff method?

(c) Does instant runoff satisfy the Condorcet winner criterion? Use your answers to parts (a) and (b) to explain how you know.

Putting It All Together

Question 3.22.* Summarize what you've learned about voting systems for more than two candidates by completing the following table. In this table, we've used the abbreviation "MC" for the majority criterion. For the entries in the table, by "Yes" we mean the voting system labeling the row of the entry satisfies the property labeling the column, and by "No" we mean the voting system labeling the row violates the property labeling the column.

	Anonymous	Neutral	Monotone	MC	CWC
Plurality	Yes	Yes	Yes	Yes	No
Borda Count					
Seq. Pairwise					
Instant Runoff					

Question 3.23.* Consider again the CVAAB presidential election from Question 2.8. Who would win the election under the instant runoff method?

Question 3.24. Write a professional letter to the editor of *Squeaks and Squawks* (the official newsletter of the CVAAB) expressing your opinion about which voting system you think should be used to elect the next president of the CVAAB. Use the results of your investigations in this and the previous chapter to give a convincing argument as to why you think the system you've chosen is the most reasonable and would best represent the will of the voters. Be sure to discuss the pros and cons of your proposed system, carefully comparing and contrasting it to the other potential options.

Questions for Further Study

Question 3.25. Answer *true* or *false* for each of the following statements, and give a convincing argument to justify each of your answers.

(a) If a voting system satisfies the Condorcet winner criterion, then it must also satisfy the Condorcet loser criterion.

(b) If a voting system satisfies the Condorcet loser criterion, then it must also satisfy the Condorcet winner criterion.

Question 3.26. Consider again the election for the PU Mathematics Department Chair from Question 3.17.

(a) Who would win the election under the Borda count?

(b) In light of your answers to Question 3.17 and part (a) of this question, who do you think should be declared the winner of the election? Give a convincing argument to justify your answer.

(c) Is there an agenda under which Amaya would win using sequential pairwise voting? What about Carlos? Delilah? Explain your answers.

Question 3.27. In part (f) of Question 3.7, you explained why the Borda count violates the Condorcet winner criterion without providing an actual example to illustrate this fact. Construct an actual example (one involving a preference schedule) to show that the Borda count can violate the CWC.

Question 3.28. Does the Borda count satisfy the Condorcet loser criterion? If so, explain why. Otherwise, construct an example (one involving a preference schedule) to show that the Borda count can violate the CLC.

Question 3.29. Does instant runoff satisfy the Condorcet loser criterion? If so, explain why. Otherwise, construct an example (one involving a preference schedule) to show that instant runoff can violate the CLC.

Question 3.30. Write a short biography of the Marquis de Condorcet, including his most important contributions both inside and outside of voting

theory, some information about his political importance in the French Revolution, and some information about his death.

Question 3.31. Write a short biography of Thomas Hare, including his most important contributions both inside and outside of voting theory.

Question 3.32. Investigate the arguments presented on the web site of FairVote (`http://www.fairvote.org`) in favor of the instant runoff voting system. Write a summary of your findings, including a comparison of the arguments presented there with our investigations in this chapter.

Question 3.33. Investigate the results of the 1991 Louisiana gubernatorial election, and write a summary of your findings. Include in your summary a brief description of the three most prominent candidates, their platforms and personal situations, and the eventual winner of the election. Also describe whether you think there could have been a Condorcet winner and/or loser in the election, and which candidate you think best represented the will of the voters. Clearly explain your reasoning, and use actual data from the election to help justify your claims.

Question 3.34.

(a) Find out how voting is conducted to determine the president of France, and write a summary of your findings. Include in your summary a detailed description of the voting system used to declare the winner, and how this system differs slightly from one of the systems we studied in this chapter.

(b) Investigate the results of the 2017 French presidential election, and write a summary of your findings. Include in your summary a brief description of the four candidates who received the largest numbers of votes, the platforms and personal situations of these candidates, and the eventual winner of the election.

(c) Investigate the results of the 2002 French presidential election, and write a summary of your findings. Include in your summary a brief description of the three candidates who received the largest numbers of votes, the platforms and personal situations of these candidates, and the eventual winner of the election.

Question 3.35.

(a) Find out how voting is conducted to determine the host city for the Olympic games, and write a summary of your findings. Include in your summary a description of how the nominees are selected, who votes, how the voting is conducted, the voting system that is used to declare the winner, and how this system differs slightly from one of the systems we studied in this chapter.

(b) Investigate the results of the voting that was held by the International Olympic Committee (IOC) to determine the host city for the

2012 Summer Olympic Games from the five finalists. Write a summary of your findings, including the finalists, winner, and the results from each round of voting.

(c) Investigate the results of the voting that was held by the IOC to determine the host city for the 2000 Summer Olympic Games from the five finalists. Write a summary of your findings, including the finalists and winner, the results from each round of voting, and the reason why one voter chose to abstain from voting in the last two rounds.

Question 3.36. Investigate the process through which candidates are nominated to receive one of the major Academy Awards (the "Oscars"), and write a detailed summary of your findings. (Note: There are two stages in the process for being selected to receive an Academy Award—the nomination stage and the final balloting to determine the winner. We are not asking for a description of the final balloting; this is done by plurality. We are asking for a description of the *nomination* stage.)

Question 3.37. Investigate Coombs' voting system, and write a summary of your findings. Include in your summary a description of how the system works, the similarities and differences between Coombs' system and the other voting systems we have studied, which of the criteria we have discussed for evaluating voting systems (anonymity, neutrality, monotonicity, the majority criterion, the CWC, and the CLC) Coombs' system satisfies, and which it violates.

Question 3.38. Research the voting system used by the reality TV show *Survivor*, and write a detailed summary of your findings. Include in your summary a comparison of this voting system to the other systems we have investigated (including Coombs' system from Question 3.37), and an analysis of the voting system according to the fairness criteria we have developed.

Question 3.39. Come up with a preference schedule for an election with four candidates in which plurality, the Borda count, sequential pairwise voting (with some agenda you specify), and instant runoff would all yield different outcomes, *and* for which the outcome of one of these four methods would demonstrate a violation of the Condorcet loser criterion.

Question 3.40. For each of the following statements, give a brief argument either for or against the statement. Support your argument with a specific example, such as a preference schedule that illustrates or refutes the claim given in the statement.

(a) The Borda count is easier to manipulate than instant runoff voting.

(b) The Borda count is less likely than other methods to elect a *consensus* or *compromise* candidate—that is, a candidate who is acceptable to a large portion of the electorate (even if relatively few voters rank them first).

(c) In certain situations, it is possible that no matter what voting method is used, and no matter what candidate is elected, some other candidate will be preferred by a substantial majority of the voters.

(d) There are situations in which the Condorcet winner may not be the most socially desirable outcome.

(e) Plurality is more likely than other voting systems to elect a Condorcet loser.

Question 3.41. Go to FairVote's web site (`http://www.fairvote.org`), and search for "instant runoff and monotonicity." You should find a page that argues that the potential for instant runoff to violate monotonicity is not a fatal flaw. Summarize and critique this argument.

Question 3.42. In 2016, voters in the state of Maine passed a referendum that would have made Maine the first state in the U.S. to adopt instant runoff for gubernatorial elections and for congressional elections at both the federal and state levels. Research some of the legal drama that affected the implementation of the law, and write a detailed summary of your findings. Has Maine actually implemented instant runoff voting yet? (At the time this book was written, the answer was no.)

Answers to Starred Questions

3.3. Assuming the data in Table 3.1 is correct, Norm Coleman was a Condorcet winner and Jesse Ventura was a Condorcet loser.

3.4. In head-to-head contests, A would beat B, B would beat C, and C would beat A, each by a vote of 2 to 1. This is indeed strange, since we would normally assume that if A beats B and B beats C, then A should beat C. There is neither a Condorcet winner nor a Condorcet loser.

3.5. If an election had two different Condorcet winners, say candidates A and B, then in a head-to-head contest between A and B, A would have to beat B *and* B would have to beat A. Since this obviously could never occur, it is impossible for an election to have more than one Condorcet winner.

3.7. (a) If majority rule does not result in a tie, then there must be one candidate in the election who is ranked first by more than half of the voters. Even without the votes of any of the other voters in the election, this candidate would win a head-to-head contest against any of the other candidates.

(b) Part (a) does *not* imply that majority rule satisfies the Condorcet winner criterion. To construct an example showing that majority rule can violate the CWC, consider an election with three candidates in which there is a Condorcet winner, but no candidate receives a majority of the first-place votes.

(c) Part (a) does imply that majority rule satisfies the Condorcet loser criterion, since a Condorcet loser can never receive a majority of the first-place votes. (Can you explain why this is true?)

(d) Think back to the type of elections we looked at in Chapter 1.

(e) Part (a) establishes that if a candidate in an election receives a majority of the first-place votes, then that candidate will be a Condorcet winner. Thus, if a voting system ever fails to elect a majority winner, it will have also failed to elect a Condorcet winner.

(f) Since the Borda count violates the majority criterion, it must, by part (e), violate the CWC.

3.9. (a) In Step 1, Gerald would win. He would then lose to Filiz in Step 2, who would then lose to Ivan in Step 3. Thus, Ivan would win the election.

(b) Based on your answer to part (a), you might be tempted to say that the best societal preference order would be $I \succ F \succ G \succ H$. However, you might want to consider whether Ivan would beat Gerald in a head-to-head contest.

(c) Ivan being elected does seem strange, since he was ranked last in the societal preference orders produced by both plurality and the Borda count.

(d) There is neither a Condorcet winner nor loser.

3.10. A Condorcet winner, by definition, could never lose a head-to-head contest with another candidate. Thus, a Condorcet winner, if one exists, will always advance to the next round in sequential pairwise voting and ultimately be declared the winner. From this we can conclude that sequential pairwise voting satisfies the Condorcet winner criterion.

3.12. Helen would win.

3.17. Delilah would be eliminated first, followed by Brandon and then Carlos. Thus, Amaya would win the election, and the resulting societal preference order would be $A \succ C \succ B \succ D$.

3.19. With these new preferences, Brandon would win the election. This implies that instant runoff is not monotone (!), since a change favorable only to Amaya on individual preference ballots would cause her to go from winning the election to losing.

3.22. The table can be completed as follows:

	Anonymous	Neutral	Monotone	MC	CWC
Plurality	Yes	Yes	Yes	Yes	No
Borda Count	Yes	Yes	Yes	No	No
Seq. Pairwise	Yes	No	Yes	Yes	Yes
Instant Runoff	Yes	Yes	No	Yes	No

3.23. Helen would win.