Introduction to Random Numbers and The Monte Carlo Method

Problem 1 *Consider the graph of the function $f(x)$ shown in figure 1. Show that $f(x)$ is a PDF. Calculate the probability that a random variable is between 3 and 8.*

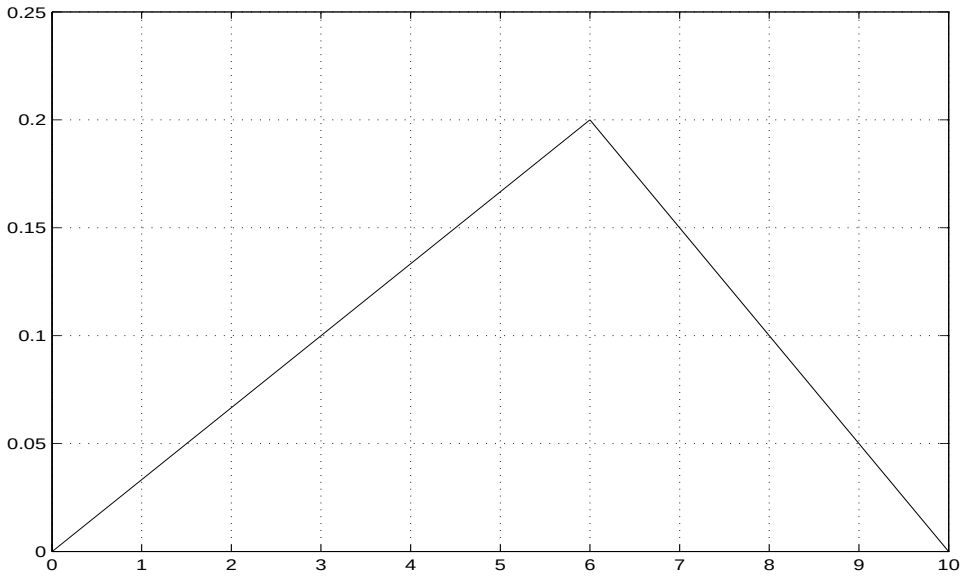

Figure 1: Example of a simple PDF.

Problem 2 *Generate a vector containing 1000 random entries from a normal distribution. Make a histogram plot of these random numbers. How is the histogram plot related to the PDF?*

Problem 3 *As mentioned in the reading, we will design a* MATLAB *experiment to simulate the dart board experiment, and hence compute an approximation of π. First, the following code can be use to construct the dart board:*

```
figure
plot([-1,1,1,-1],[-1,-1,1,1])
plot([-1,1,1,-1,-1],[-1,-1,1,1,-1])
axis([-1.5,1.5,-1.5,1.5])
axis square
hold on
theta = linspace(0,2*pi,200);
plot(cos(theta), sin(theta),'r')
```

We now want to generate random dart throws at the board. That is, we want to generate random points (x, y) with $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. We can do this (for, say, 100 dart throws) as follows:

```
x = 2*rand(100,1) - 1;
y = 2*rand(100,1) - 1;
```

If we want to visualize where these dart throws hit the board, we simply plot the points:

```
plot(x, y, 'o')
```

We can now get an estimate of π using the formula given in the reading:

```
NumberInside = sum(x.^2 + y.^2 <= 1);
PiEstimate = 4 * (NumberInside / 100)
```

Since only 100 random throws were used, it is unlikely that this estimate for π will be very accurate. Better accuracy can be obtained by increasing the number of throws.

Try increasing the number of throws to 1000, 10000 and 100000. What is the estimated value of π that you compute? What do you conclude about using a Monte Carlo method for computing an approximation of π?

Problem 4 *The area under a marginal cost (MC) curve from point a to point b is the cost to increase production from a units to b units. Given the $MC(q) = 0.06q^2 - q + 10$, use the Monte Carlo Method to find the cost to increase production from 15 units to 20 units. (The area under a curve means the area from the curve to the x-axis).*