

Section 4.7  
Logistic Growth

1. If  $t$  is in years since 1990, one model for the population of the world,  $P$ , in billions, is
$$P = \frac{40}{1+11e^{-0.08t}}.$$
  - a. What does this model predict for the maximum sustainable population of the world?
  - b. Sketch the graph of  $P$  against  $t$ .
  - c. According to this model, when will the earth's population reach 20 billion? 39.9 billion?
  
2.
  - a. Draw a logistic curve. Label the carrying capacity  $L$  and the point of diminishing returns  $t_0$ .
  - b. Draw the derivative of the logistic curve. Mark the point  $t_0$  on the horizontal axis.
  - c. Suppose a company keeps track of the rate of sales (for example, sales per week) rather than total sales. Explain how the company can tell on a graph of rate of sales when the point of diminishing returns is reached.

3. The Metro Department Store found that  $t$  wk after the end of a sales promotion the volume of sales was given by a function of the form  $S(t) = B + Ae^{-kt}$  where  $0 \leq t \leq 4$  where  $B = 50,000$  and is equal to the average weekly volume of sales before the promotion. The sales volumes at the end of the first and third weeks were \$83,515 and \$65,055, respectively. Assume that the sales volume is decreasing exponentially.
- Find the decay constant  $k$ .
  - Find the sales volume at the end of the fourth week.
  - How fast is the sales volume dropping at the end of the fourth week?