## **Block Matrix Operations**

Goals: Get more comfortable with block matrices, practice multiplying block matrices and block matrix-vector multiplication

In general, if 
$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$
 and  $B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$  then  $AB = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix}$ .

1. Calculate AB.

$$A = \begin{pmatrix} 1 & 2 & | & 0 & 0 \\ 0 & 1 & | & 0 & 0 \\ \hline 0 & 0 & | & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & | & 0 \\ -1 & 1 & | & 0 \\ \hline 0 & 0 & | & 1 \\ 0 & 0 & | & 3 \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}$$

- 2. Calculate  $A\mathbf{x}$  and  $B\mathbf{x}$  if they make sense.
- 3. The matrix is in block lower triangular form if it looks like:  $A = \begin{pmatrix} B & 0 \\ C & D \end{pmatrix}$ . What are blocks B and D for A1?

$$A1 = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 5 & 6 & 0 & 0 \\ 3 & 4 & 9 & 10 \\ 7 & 6 & 11 & 12 \end{pmatrix}$$

- 4. Find the projection matrix for the graph in figure 1.
- 5. What type of matrix is it? How can you tell?



Figure 1: Graph for Geographically subdivided population.