## Block Matrix Operations

Goals: Get more comfortable with block matrices, practice multiplying block matrices and block matrix-vector multiplication

In general, if $A=\left(\begin{array}{ll}A_{11} & A_{12} \\ A_{21} & A_{22}\end{array}\right)$ and $B=\left(\begin{array}{ll}B_{11} & B_{12} \\ B_{21} & B_{22}\end{array}\right)$ then $A B=\left(\begin{array}{ll}A_{11} & A_{12} \\ A_{21} & A_{22}\end{array}\right)\left(\begin{array}{ll}B_{11} & B_{12} \\ B_{21} & B_{22}\end{array}\right)=$ $\left(\begin{array}{ll}A_{11} B_{11}+A_{12} B_{21} & A_{11} B_{12}+A_{12} B_{22} \\ A_{21} B_{11}+A_{22} B_{21} & A_{21} B_{12}+A_{22} B_{22}\end{array}\right)$.

1. Calculate $A B$.

$$
A=\left(\begin{array}{cc|cc}
1 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\hline 0 & 0 & 2 & 1
\end{array}\right) \quad B=\left(\begin{array}{cc|c}
1 & 2 & 0 \\
-1 & 1 & 0 \\
\hline 0 & 0 & 1 \\
0 & 0 & 3
\end{array}\right) \quad \mathbf{x}=\left(\begin{array}{c}
1 \\
-1 \\
0 \\
3
\end{array}\right)
$$

2. Calculate $A \mathbf{x}$ and $B \mathbf{x}$ if they make sense.
3. The matrix is in block lower triangular form if it looks like: $A=\left(\begin{array}{cc}B & 0 \\ C & D\end{array}\right)$.

What are blocks B and D for $A 1$ ?

$$
A 1=\left(\begin{array}{cccc}
1 & 2 & 0 & 0 \\
5 & 6 & 0 & 0 \\
3 & 4 & 9 & 10 \\
7 & 6 & 11 & 12
\end{array}\right)
$$

4. Find the projection matrix for the graph in figure 1.
5. What type of matrix is it? How can you tell?


Figure 1: Graph for Geographically subdivided population.

