

# ***ONLINE EDUCATION, SIGNALING, AND HUMAN CAPITAL***

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## *Abstract*

Online higher education may lower educational time cost for less able individuals more than for others. If education merely signals ability, decreasing education cost for the less able may *decrease* welfare by increasing over-investment in education by the more able. When education adds to human capital *and* may signal ability, decreasing education cost for the less able is *more* likely to increase welfare the *smaller* the productivity difference between the more and less able, and the *smaller* the fraction of the more able in the population.

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## 1. Introduction

The increased use of online education could lead to significant changes in higher education (Acemoglu, Laibson, and List, 2014). There are many issues involving online versus traditional (face-to-face) higher education. One question is which method is better for learning. That issue has not been resolved. Some find worse performance in online courses versus traditional courses, some find the opposite, and some find no performance difference between the two delivery methods.<sup>1</sup>

An additional issue is whether online courses can lower time costs for students. Online courses have no commuting costs, and there is less repetition as students control what they watch (Cowen and Tabarrok, 2014). Students can study when they have peak energy, and can manage their time and complete assignments on their own schedules (OEDb, 2012, and University of Washington, 2013).

Two studies provide evidence of reduced time costs with online courses. First, Bowen *et al.* (2013) examine statistics classes at six public universities in 2011. Some students were randomly assigned to take a hybrid version of the course in which they had one hour per week of face-to-face instruction along with computer-guided instruction. Others took the traditional course with three to four hours of face-to-face instruction per week. On average, students in the hybrid course spent about 25% less total time on the course than did those in the traditional

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<sup>1</sup> The US Department of Education (DOE) released a long report (Means *et al.*, 2010) which concluded that online education leads to improved student outcomes. However, the DOE meta-analysis mainly focused on studies in which students were not randomly assigned to online or traditional courses. Also, with two limited exceptions, the studies did not involve the same instructor teaching online and traditional courses (Figlio *et al.*, 2013). Navarro and Shoemaker (2000) found superior performance in online introductory macroeconomic courses, and Brown and Liedholm (2002) found the opposite in introductory microeconomic classes. In the latter two studies, the traditional courses were not comparable in terms of additional resources available to students (Figlio *et al.*, 2013). Figlio *et al.* find some evidence of negative effects of online instruction for low achieving students when the online course essentially involved videotaped lectures. Bowen *et al.* (2014) find no such negative effects comparing a hybrid (some face-to-face instruction) online course with a traditional course when the online course was more interactive than in the course analyzed by Figlio *et al.*

course. Note that students in the two different kinds of courses had no statistically significant differences in learning outcomes such as pass rates and scores on common exam questions.

A second study is based on statistics courses at Carnegie Mellon University in 2005 and 2006. Students volunteered to participate in an online only class with no instructor.<sup>2</sup> Some of these volunteers were randomly chosen to participate in the online course. These students performed as well or better than those in the traditional class (not controlling for demographic characteristics), and did so in about 50% of the time<sup>3</sup> (Lovett *et al.*, 2008).

The evidence does not suggest whether online education reduces time cost by a relatively larger amount for certain students. Posner (2012) notes how online classes have advantages for students who cannot work as fast as others. The ability to watch lectures online can reduce the total time costs for less able individuals more than for others because the former individuals spend less time trying to grasp material when they can watch a lecture at their own pace.<sup>4</sup>

If education only adds to human capital, lowering cost for the less able benefits them and has no effect on others. Conversely, education may be used as a signal of (pre-matriculation) ability (Spence, 1974, 2002).<sup>5</sup> Suppose education is only a signal, and does not affect productivity. Then, the lower the level of education, the greater is social welfare. Since the level of the signal is inversely related to the educational cost difference between more and less productive individuals, *reducing* this cost difference decreases welfare.

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<sup>2</sup> The hybrid statistics course in the study by Bowen *et al.* (2013) is derived from the same materials used for the online only course at Carnegie Mellon.

<sup>3</sup> The online class was 8 weeks versus 15 weeks for the traditional class, and time-use surveys indicate that students spent about the same amount of time per week in either type of class.

<sup>4</sup> I ignore problems with online classes. Haynie (2014) argues that, at least for large scale online classes, completion rates are not good. Banerjee and Duflo (2014) note that only 5% of students enrolled in a large online graduate education course at the University of Pennsylvania completed the class.

<sup>5</sup> In a study for the Social Science Research Council, Arum *et al.* (2011) find that 36% of U.S. college students learn very little after four years. One economist (Caplan, 2011) recently accepted the idea that little is learned in college, higher education is essentially a signal, and welfare would be improved if students spent fewer years in school.

I analyze a theoretical model in which education increases productivity *and* serves as a signal. My goal is to examine the effects on welfare of changing the educational time cost of the less able relative to that of the more able. Well before the advent of online education, Riley (1981) considered how lowering education cost for the less able could *lower* welfare:

Consider "...the adoption of an innovation which increases the rate of educational advancement of the less able...The higher rate of educational advancement implies a reduction in the marginal time costs of education...and hence an increase in the education of..." the less able. The more able "...must *increase* their education...in order to be differentiated."<sup>6</sup>

More recently, McAfee *et al.* (2015) suggests that the best subjects for signaling are those that are less useful or practical since they may imply the *biggest* cost difference between more and less able individuals:

"...interpreting long medieval poems could more readily signal the kind of flexible mind desired in management than studying accounting, not because the desirable type is good at it, or that it is useful, but because the less desirable type is so much worse at it."<sup>7</sup>

The idea of analyzing medieval poems suggested by McAfee *et al.* as a good signal is actually supported by some evidence. Bukszpan (2012) reports on the value of seemingly useless degrees. One individual majored in epic Renaissance literature, and works as a financial analyst. She claims her critical skills in analyzing literature are important in making smart investment choices. Of course, her education may have added to her analytical skills. However, it may also be true that some of what potential employers learned from her major is that she had the analytical skills required to master such a subject. That is the signaling role of education.

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<sup>6</sup> Riley (1981), p.375.

<sup>7</sup> McAfee *et al.* (2015), p.246.

I assume education adds to productivity and signals inherent ability.<sup>8</sup> I develop a model in Section Two that enables me to consider the effect on welfare of changing education cost for the less able. Online education may lower the cost of education for the less able relative to the more able. Alternatively, more rigorous courses may increase the cost of education for the less able relative to the more able.

In Section Three, I consider numerical examples to provide further insight on the value of changing education cost for the less able. I summarize my findings in Section Four.

## 2. A model of productive education

Consider a world where there are two types of individuals,  $A$ s and  $B$ s. Education is denoted by  $y$ . The length of potential work life is set at one, and discounting is ignored. Education cost is simply time cost (Riley, 1976, 1979b, and Weiss, 1983). An  $A$  gets  $y$  units of education by investing for the fraction of time  $y$ . A  $B$  gets  $y$  units of education by investing for the fraction of time  $zy$ ,  $z > 1$ . Employers observe only  $y$ .<sup>9</sup> Productivity depends on one's type and education. Productivity for an  $A$  is  $\theta y$  (received for  $1-y$  of an  $A$ 's work life), with  $\theta > 1$ . Productivity for a  $B$  is  $y$  (received for  $1-zy$  of a  $B$ 's work life). Thus, a larger  $\theta$  implies a larger inherent productivity difference between  $A$ s and  $B$ s. For simplicity, I assume education cost can be changed for  $B$ s only. Thus,  $z$  can be increased, possibly by more abstract courses, or decreased, possibly by increased online education.

With perfect information, an  $A$  would maximize  $\theta y(1-y)$ , and a  $B$  would maximize

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<sup>8</sup> Clark and Martorell (2014) consider the additional value of receiving a high school diploma, and argue there is little support for the signaling role of education. Their results do not exclude the possibility that *the level of education* (years of schooling, grade, etc.) affects productivity *and* serves as a signal of pre-matriculation ability.

<sup>9</sup> Riley (1979b) assumed educational quality is acquired more cheaply for the more able, but years of education are what is observable. I assume years of education completed,  $y$ , are observed, but total time devoted to obtaining education is not observed.

$y(1-zy)$ , implying that  $A$ s would set  $y_A = 1/2$ , and  $B$ s would set  $y_B = \frac{1}{2z}$ . Welfare would then be  $\frac{\theta}{4}$  for an  $A$  and  $\frac{1}{4z}$  for a  $B$ . I normalize the number of individuals to one, and let  $\alpha$  equal the fraction of  $A$ s in the population with  $\alpha$  known to all. Total welfare is denoted by  $W$ . Welfare for an  $A$  is denoted by  $W_A\{y_A\}$ , and welfare for a  $B$  is denoted by  $W_B\{y_B\}$ . Thus,

$$W = \alpha W_A\{y_A\} + (1-\alpha)W_B\{y_B\}.$$

A  $B$  will not mimic an  $A$  to get paid  $\theta y$  for values for  $y$  for which  $\theta y(1-zy) < \frac{1}{4z}$ . Following Riley (1979a), the lowest level of  $y$  that will allow a signaling/separating equilibrium is the Riley outcome<sup>10</sup> for  $A$ s,  $y = y_{Riley}$ .  $B$ s will not mimic  $A$ s for any  $y > y_{Riley}$ . For simplicity, I assume an indifferent  $B$  will not mimic an  $A$ . Thus,  $y_{Riley}$  is obtained by setting  $\theta y(1-zy) = \frac{1}{4z}$ . I then have:

$$y = \frac{1 \pm \left(\frac{\theta-1}{\theta}\right)^{1/2}}{2z}. \tag{1}$$

The smaller root of  $\frac{1 \pm \left(\frac{\theta-1}{\theta}\right)^{1/2}}{2z}$  is less than the perfect information level of  $y$  for a  $B$ . Thus, the lowest level of  $y$  that induces a  $B$  not to mimic an  $A$ ,  $y_{Riley}$ , is:

$$y_{Riley} = \frac{1 + \left(\frac{\theta-1}{\theta}\right)^{1/2}}{2z}. \tag{2}$$

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<sup>10</sup> The Riley outcome is when less able individuals set  $y$  equal to the level they would choose with perfect information, and more able individuals set  $y = y_{Riley}$ ---the lowest level of the signal that induces a signaling equilibrium (Riley, 1979a). Using the *intuitive criterion* (Cho and Kreps, 1987), signaling only occurs at the Riley outcome.

Note that  $\frac{\partial y_{Riley}}{\partial z} < 0$  and  $\frac{\partial y_{Riley}}{\partial \theta} > 0$ . A higher  $z$  means a higher education cost for a  $B$ .

Thus, the required education level for an  $A$  with signaling,  $y_{Riley}$ , is reduced as  $z$  increases. A larger  $\theta$  means higher earnings for an  $A$ , which makes it more worthwhile for a  $B$  to mimic an  $A$ . Thus,  $y_{Riley}$  is positively related to  $\theta$ .

With perfect information,  $A$ s set  $y = 1/2$ . Thus, if  $y_{Riley} > 1/2$ , excessive investment in education by  $A$ s is necessary to induce  $B$ s not to mimic  $A$ s. However, if education cost of the less able is high enough,  $y_{Riley} \leq 1/2$ . Then the perfect information level of  $y$  for an  $A$ ,  $1/2$ , is sufficiently large that it will deter a  $B$  from setting this level.

Using eq.(2),  $y_{Riley} > 1/2$  if:

$$z < 1 + \left(\frac{\theta-1}{\theta}\right)^{1/2} \equiv z_{Riley}. \quad (3)$$

If  $z \geq z_{Riley}$ , both types of individuals obtain the levels of  $y$  they would with perfect information. Note, the maximum value of  $\left(\frac{\theta-1}{\theta}\right)^{1/2}$  is when  $\theta \rightarrow \infty$  and  $\left(\frac{\theta-1}{\theta}\right)^{1/2} \rightarrow 1$ .

Thus,  $z_{Riley} \leq 2$ . Let welfare for an  $A$  with  $y = y_{Riley} > 1/2$  be noted by  $W_A\{y_{Riley}\}$ . Then:

$$W_A\{y_{Riley}\} = \theta y_{Riley}(1-y_{Riley}) = \frac{1}{4z^2} \left\{ 2\theta[z-1] \left[ 1 + \left(\frac{\theta-1}{\theta}\right)^{1/2} \right] + 1 \right\}. \quad (4)$$

*Proposition One. If more able individuals overinvest in education to signal their ability, their welfare increases if educational cost for less able individuals increases.*

*Proof.* See the Appendix. An increase in  $z$  lowers  $y_{Riley}$ , but, from inspection of eq.(4), does not unambiguously raise  $W_A\{y_{Riley}\}$ . A lower  $y_{Riley}$  actually can lower welfare for an A because of the direct effect of  $y_{Riley}$  on  $W_A\{y_{Riley}\}$ . However, this does not occur when  $y_{Riley} > 1/2$ . Intuitively, increasing  $z$  and lowering  $y_{Riley}$  towards  $1/2$  should increase  $W_A\{y_{Riley}\}$ , and it does.

Assuming  $y_{Riley} > 1/2$ , total welfare is  $W = \alpha W_A\{y_{Riley}\} + \frac{1-\alpha}{4z}$ , since welfare for a B who sets  $y = \frac{1}{2z}$  is  $\frac{1}{4z}$ . Maximizing  $W$  with respect to  $z$ , I have (using eq.(4)):

$$\frac{\partial W}{\partial z} = \frac{1}{4z^3} \left\{ 2\alpha \left[ \theta [2-z] \left[ 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2} \right] - 1 \right] - (1-\alpha)z \right\} \equiv \frac{J}{4z^3}. \quad (5)$$

For an interior solution for  $z$ ,  $J = 0$ . If  $J = 0$ ,  $\frac{\partial^2 W}{\partial z^2} < 0$ , so there is a maximum of  $W$  with respect to  $z$ .

*Proposition Two.* Assuming more able individuals overinvest in education in a signaling equilibrium, if these individuals are a smaller fraction of the population ( $d\alpha < 0$ ), or if their innate skills are relatively less valuable ( $d\theta < 0$ ), then the level of education cost for the less able that maximizes total welfare is decreased.

*Proof.* See the Appendix. A decrease in  $z$  lowers educational cost for Bs, raising their welfare. However, a decrease in  $z$  increases  $y_{Riley}$ , lowering welfare for As. If As are a smaller percentage of the population ( $\alpha$  is smaller), or if the output per unit of education of As is lower ( $\theta$  is smaller), there is less of a loss from moving As farther away from their welfare-maximizing level of  $y$  as  $y_{Riley}$  increases when  $z$  decreases. Thus,  $\frac{dz}{d\theta}$  and  $\frac{dz}{d\alpha}$  are both positive: the value of  $z$  that



maximizes total welfare is lower if either  $\theta$  or  $\alpha$  is reduced. Denote the value of  $z$  that maximizes total welfare,  $W$ , by  $z^*$ . Note that  $z^*$  maximizes  $W$  only when  $A$ s set  $y > 1/2$ , and  $B$ s set  $y = \frac{1}{2z}$ .<sup>11</sup>

Using Proposition Two, consider programs where the better students are a relatively small percentage of all students ( $\alpha$  is smaller), or where students are more homogeneous in ability ( $\theta$  is smaller). Then it is more likely welfare will be increased by a shift towards online courses that lowers education time cost for the less able relative to that for the more able. One caveat is if online courses are inferior for learning. As discussed in Section One, the evidence is mixed regarding learning in online versus traditional courses.

Apparently students prefer online courses that are easier, and prefer traditional delivery for more interesting courses (Fain, 2013). One implication of that finding would be that more difficult courses may not be the best candidates for online courses. However, two of the studies discussed in Section One (Lovett *et al.*, 2008, and Bowen *et al.*, 2013) involves statistics courses, which are generally not viewed as easy. In these studies, student performance in either a hybrid or a complete online course is at least as good as in a traditional course. Thus, it is not clear which subjects may be better suited for online versus traditional delivery.

I now consider a pooling equilibrium because, if  $A$ s prefer pooling to a signaling equilibrium, both  $A$ s and  $B$ s would obtain the same level of education. Since empirically different levels of education are observed, it is useful to consider when  $A$ s will *not* prefer pooling.

With pooling, all would be paid a wage equal to expected productivity, which equals

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<sup>11</sup> Weiss (1983) has a model in which education is a signal, and more education means a higher level of individual productivity. In his model, an individual receives a “grade” of pass or fail in addition to accumulating units of education. In that case, the more able may *under-invest* in education because they choose the minimum amount of education required to pass the test. In my model, there is no grade, and too little investment in education by the more able does not occur.

$(\theta\alpha + 1 - \alpha)y$  for  $1 - y$  of an  $A$ 's work life. An  $A$  would then choose  $y$  to maximize

$(\theta\alpha + 1 - \alpha)y(1 - y)$ , yielding  $y = 1/2$ . Thus, pooling involves both types setting  $y = 1/2$ , the perfect information level of  $y$  for an  $A$ , so  $B$ s over-invest in education in a pooling equilibrium.

As Spence (2002) noted, if the marginal education cost for the less able ( $B$ s herein) is too high, then they will not choose to mimic the more able at the perfect information level of education for the latter. Herein, that occurs if  $z > z_{Riley}$ . Let  $\psi \equiv \alpha\theta + 1 - \alpha$ , and note that

$$z_{Riley} \equiv 1 + \left(\frac{\theta - 1}{\theta}\right)^{1/2}. \text{ I then have:}$$

*Proposition Three. The more able prefer to over-invest in education rather than pool with the less able if education cost for the less able is sufficiently large, or if*

$$z > \hat{z} \equiv \frac{\theta z_{Riley} - [\theta z_{Riley} (\theta z_{Riley} - 2\psi) + \psi]^{1/2}}{\psi}.$$

*Proof.* See the Appendix. For  $z < \hat{z}$ ,  $y_{Riley}$  is large enough that  $A$ s prefer to pool with  $B$ s at  $y = 1/2$ .

Since  $\frac{\partial y_{Riley}}{\partial z} < 0$ , if  $\hat{z} < z$ ,  $A$ s prefer over-investment in education ( $y = y_{Riley} > 1/2$ ) to pooling with

$B$ s at  $y = 1/2$ .<sup>12</sup> I do not prove that  $\hat{z} < z_{Riley}$ , but, in all of the cases I consider, this inequality

holds. Thus, using Proposition Three, there is a range of  $z$ ,  $\hat{z} < z < z_{Riley}$ , in which a signaling

equilibrium will occur, with  $y = y_{Riley} > 1/2$  chosen by  $A$ s.

Suppose education is not productive so productivity of  $A$ s and  $B$ s is independent of  $y$ .

Then the pooling equilibrium in which both types set  $y = 0$  implies the highest level of welfare. If education could be taxed (Spence, 2002), welfare would improve.

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<sup>12</sup> Mailath *et al.* (1993) use the idea of *undefeated equilibrium* to find when a pooling equilibrium would be broken by a signaling equilibrium, that is, when signaling is preferred to pooling by the more able. The *intuitive criterion* (Cho and Kreps, 1987) rules out all pooling equilibria in such situations. The *undefeated equilibrium* refinement essentially amends the *intuitive criterion* to allow for a pooling equilibrium when pooling implies higher welfare for more able individuals than would a separating equilibrium with  $y = y_{Riley}$ . Undefeated equilibrium effectively allows commitment to wage offers by firms to be endogenous (Koufopoulos, 2011, Perri, 2014). I follow Mailath *et al.*, and assume a signaling equilibrium will only result if it is preferred to pooling by more able individuals.

In the model herein, education is productive. I cannot generally compare the different equilibria because they depend on the value of  $z$ . Given  $z$ , welfare is always highest when each type sets  $y$  at the level each would choose under perfect information---  $y = \frac{1}{2z}$  for  $B$ s, and  $y = \frac{1}{2}$  for  $A$ s.<sup>13</sup>

### 3. Analysis.

#### A. Numerical values versus the evidence

In Table One, I show critical values for  $z$  for given values of  $\theta$  and  $\alpha$ . In Table Two, I show total wealth,  $W$ , for  $\alpha$  of either .2 or .4, and  $\theta$  of 1.5, 2, and 3. I will discuss the results in Table Two in sub-section B.

For brevity, I do not discuss all the values of  $\alpha$  and  $\theta$  in Table One. Also, as seen in Table One, high values of  $\alpha$  are more likely to result in a pooling equilibrium (because  $\hat{z}$  is larger so it is more likely that  $z < \hat{z}$ ) in which both types of individuals obtain the same amount of education. Since I wish to focus on possible over-investment in education by the more able, I use relatively low values of  $\alpha$ . Using  $\alpha$  of either .2 or .4, pooling does not occur with the values of  $\theta$  I use if  $z \geq 1.2$ .

If pooling does occur, lowering education cost for the less able ( $dz < 0$ ) will necessarily increase welfare. With pooling,  $B$ s set  $y = \frac{1}{2}$ , that is, they over-invest in education relative to a world of perfect information. All that happens with pooling as  $z$  decreases is  $B$ s' welfare

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<sup>13</sup>For example, at low enough education cost for the less able,  $z < \hat{z}$ , pooling occurs at  $y = \frac{1}{2}$ . It can be shown that  $B$ s are better off than with perfect information, but  $A$ s are worse off since they earn less. Since  $B$ s over-invest in education, welfare is lower than under perfect information--- $A$ s lose more than  $B$ s gain since the former earn less with pooling than with perfect information---  $\frac{\theta\alpha+1-\alpha}{4}$  with pooling versus  $\frac{\theta}{4}$  with perfect information.

increases: As and Bs continue to set  $y = \frac{1}{2}$ . However, unless  $\theta$  and  $\alpha$  are relatively large,  $\hat{z}$  is relatively low (Table One), so the range of  $z$  for which pooling occurs is small.

One recent study finds, on average, lifetime earnings with a bachelor's degree are 74% higher than with a high school degree, 47% higher than with some college, and 31% higher than with an associate's degree (Carnevale *et al.*, 2011). From Table Three, when  $z$  is 1.2, and  $\theta$  is either 1.5 or 2, the results are not too far from the empirical difference between the lifetime earnings of a high school and college graduate. However, the educational difference for these two cases (58% and 71%) is less than the high school versus college difference if  $y$  is viewed as measuring education after one can leave school, and that is age 16---typically after one's sophomore year. In that case, a high school graduate has  $y = 2$ , and a college graduate has  $y = 6$ .

I compare bachelor's and associate's degrees. With  $y$  again measuring education after the sophomore year of high school, then  $y$  for an associate's degree is 4, and  $y$  for a bachelor's degree is 6, or  $\frac{y_A}{y_B} = 1.5$ . This is similar to the ratio of  $y_A$  to  $y_B$  for  $\theta$  equal to 1.5 and  $z$  equal to 1.2 and 1.5. However, the lifetime earnings differential in my theoretical model is far above that found empirically for bachelor's degree recipients versus those who earn associate's degrees. From Table Three, when  $\theta = 1.5$  and  $z = 1.2$  (which yields the lowest ratio of lifetime earnings for As relative to Bs in Table Three), the earnings advantage for those with a bachelor's degree relative to those with an associate's degree is about double what is found empirically---62% versus 30%.

Thus, there is some similarity between the numerical values based on the theoretical model herein and the empirical evidence, but there are also differences. Although the model highlights the important forces at work, one should be cautious in extrapolating from the results herein.

### *B. Changing cost for the less able*

My argument is that increased online education can decrease education cost for less able individuals ( $dz < 0$ ), and that more abstract education can increase education cost for this group ( $dz > 0$ ). In Table Two, I consider cases when there would be a separating/signaling equilibrium ( $z > \hat{z}$ ). In all of these equilibria, *Bs* choose their perfect information level of education:  $y_B = \frac{1}{2z}$ . If  $z > z_{Riley}$ , then *As* need not over-invest in education. Thus, when  $z > z_{Riley}$ ,  $y_A = 1/2$ , and, when  $z < z_{Riley}$ ,  $y_A = y_{Riley} > 1/2$  (eq.(2)).

First, with the six combinations of  $\theta$  and  $\alpha$  in Table Two, suppose  $z = 1.5$ , and can be either increased or decreased by 20%. Thus,  $z$  becomes either 1.2 or 1.8. Lowering  $z$  to 1.2 lowers welfare in two cases, raises welfare in three cases, and, in one case, welfare is unchanged as  $z$  is decreased. Raising  $z$  to 1.8 lowers welfare in four cases, and raises welfare in two cases. Thus, at least starting with  $z = 1.5$ , welfare is more likely to increase if  $z$  is decreased than if it is increased.

Second, I consider only a reduction in time cost for the less able. Suppose  $z = 1.8$ , and can be decreased by 33% to  $z = 1.2$ . In the six examples in Table Two, this reduction in cost for the less able raises total welfare in four cases, and lowers total welfare in two cases. The average of the four increases in welfare is about 8%, and the average of the two decreases in welfare is about 5%. In these six cases, decreasing  $z$  increases welfare if  $\theta = 1.5$  and  $\alpha = .2$ ,  $\theta = 2$  and  $\alpha = .2$ ,  $\theta = 3$  and  $\alpha = .2$ , and  $\theta = 1.5$  and  $\alpha = .4$ . As noted above, decreasing  $z$  is more likely to raise welfare the smaller are  $\alpha$  and  $\theta$ .

If initially  $z < z_{Riley}$ , and  $z$  is increased so that  $z > z_{Riley}$ , then it is not surprising that total welfare can increase. The reason is that *As* no longer must over-invest in education to separate

from *Bs*. For example, if  $\theta = 2$  and  $\alpha = .2$ ,  $z_{Riley} = 1.707$ . Then increasing  $z$  from 1.5 to 1.8 reduces  $y_A$  from .569 to .5. Welfare of *As* increases more than welfare of *Bs* decreases, and total welfare rises from .231 to .239.

However, even if *As* still over-invest in education as  $z$  increases, with  $\frac{\partial y_{Riley}}{\partial z} < 0$ , it is possible that an increase in education cost for the less able may increase total welfare. Again using Table Two, suppose  $\theta = 2$  and  $\alpha = .4$ . Then an increase in  $z$  from 1.2 to 1.4, an increase of approximately 17%, results in a welfare increase of about 6% (from .279 to .297). *Bs* decrease  $y$  from .417 to .357, and are worse off. *As* decrease  $y$  from .711 to .61, and are better off because they reduce (but do not eliminate) over-investment in  $y$ .

Finally, I consider the case when  $\theta = 3$  and  $\alpha = .4$ . Now increasing  $z$  from 1.2 to 1.6, an increase of approximately 33%, results in a welfare increase of about 12% (.346 to .388). Even though *As* still over-invest in education with  $z = 1.6$ , they only get about 75% of the education with  $z = 1.6$  that they get with  $z = 1.2$  (.568 versus .757).

The last two examples demonstrate that, if online courses lower education cost for the less able relative to that for the more able, welfare may decrease. As demonstrated previously welfare is likely to *increase* as  $z$  *increases* the larger are  $\theta$ , the productivity advantage for *As* relative to *Bs*, and  $\alpha$ , the fraction of *As* in the population.

#### 4. Summary

Online higher education may lower the time cost of education for less able individuals relative to that for the more able, increasing welfare if education only adds to human capital.<sup>14</sup>

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<sup>14</sup> There are many issues concerning online education. One is the importance of direct personal contact (lacking online) in education (Becker, 2012). A second issue is how much the internet will replace traditional university courses. Weissmann (2012) argues campuses involve more than teaching, and mentions the signaling value of

When education is a signal of inherent ability, possible over-investment in education by the more able may occur (Spence, 1974, 2002). When over-investment occurs, it is because it is necessary to prevent the less able from mimicking the educational choices of those who are more able. Less over-investment occurs the larger the difference in education cost between less able and more able individuals. More abstract education may result in an *increase* in the educational cost difference between less and more able individuals, reducing over-investment in education by the more able, the opposite of what may result from increased online education.

I considered the possibility that education adds to human capital *and* may be a signal of inherent ability. Then, assuming more able individuals over-invest in education in a signaling equilibrium, if these individuals are 1) a smaller fraction of the population, or if 2) the output per unit of education of the more able is lower, then a lower level of education cost for the less able is more likely to be consistent with maximization of total welfare. The loss in welfare for the more able from lowering education cost for the less able is due to increased over-investment in education by the more able. Fewer more able individuals, and a lower output per unit of education for the more able both imply a smaller decrease in welfare for them as they move further from their welfare-maximizing level of education.

These results suggest when it may be optimal to increase online education, assuming there is no reduction in the quality of education. When education involves less of a return in the labor market for the more able relative to the less able, and students tend to be less able, then the

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education. It is possible online courses provide worse signals because some of what is required in other classes, such as attendance and group projects that require personal contact, is missing. Roth (2013) suggests that elite universities will survive the online class revolution, at least in part because of the signaling that occurs at such schools. Since there is a continuum of universities in terms of quality, Roth's argument implies schools that are not elite, but that are not at the lowest end of the quality continuum, may also survive the spread of online education.

benefits of reducing cost for the less able relative to the more able are more likely to exceed the cost of doing so.

Two factors make it unlikely significant *increases* in time cost of education for the less able will increase total welfare. First, as time costs increase, the welfare loss for the less able--- due to both higher cost and the resultant lower optimal level of education---becomes larger than the gain to the more able from less over-investment in education. Second, at a high enough education cost for the less able, over-investment in education does not occur.

It appears that the signaling role of traditional and online education is a topic that deserves further attention, given the possibility of significant growth in online education. Such education may reduce educational cost differences between those with different ability levels, which may or may not increase total welfare.



## Appendix

*Proof of Proposition One.* I formally demonstrate that welfare of the more able increases as education cost for the less able increases when the more able overinvest in education ( $y_{Riley} > 1/2$ ).

Using  $W_A\{y_{Riley}\}$  from eq.(4):

$$\frac{\partial W_A\{y_{Riley}\}}{\partial z} = \frac{1}{2z^3} \left\{ \theta [2 - z] \left[ 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2} \right] - 1 \right\}. \quad (A1)$$

Since  $z < 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2}$  for  $y_{Riley} > 1/2$ , let  $z = 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2} - \varepsilon$ ,  $\varepsilon > 0$ . Then:

$$\frac{\partial W_A\{y_{Riley}\}}{\partial z} = \frac{\varepsilon \theta}{2z^3} \left[ 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2} \right] > 0. \quad (A2)$$

*Proof of Proposition Two.* I show that  $\frac{dz}{d\theta}$  and  $\frac{dz}{d\alpha}$  are both positive. Totally differentiating eq.(5):

$$\begin{aligned} \frac{1}{4z^3} \frac{\partial^2 W}{\partial z^2} dz + \frac{1}{4z^3} \left\{ 2 \left\{ \theta [2 - z] \left[ 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2} \right] - 1 \right\} + z \right\} d\alpha \\ + \frac{2\alpha}{4z^3} [2 - z] \left[ 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2} + \frac{1 \left( \frac{\theta-1}{\theta} \right)^{-1/2}}{2\theta} \right] d\theta = 0. \end{aligned} \quad (A3)$$

Using eqs.(A1) and (A2), the  $\{\bullet\}$  term in eq.(A3) is positive. Also, the  $[\bullet]$  term multiplied by  $d\theta$  in eq.(A3) is clearly positive. Since  $\frac{\partial^2 W}{\partial z^2}$  is negative for a maximum of  $W$ ,  $\frac{dz}{d\theta}$  and  $\frac{dz}{d\alpha}$  are both positive.

*Proof of Proposition Three.* I show when As prefer signaling with  $y = y_{Riley} > 1/2$  to pooling.

Let  $\psi \equiv \alpha\theta + 1 - \alpha$ , and note that  $z_{Riley} \equiv 1 + \left( \frac{\theta-1}{\theta} \right)^{1/2}$ .

Pooling earnings for an A =  $(\alpha\theta + 1 - \alpha)y$  received for 1-y of work life.

With  $y = 1/2$  with pooling, an A's welfare is then  $\frac{\alpha\theta + 1 - \alpha}{4}$ .

Welfare for an A with signaling and  $y = y_{Riley} > 1/2$  is found in eq.(4) in the text.

Setting signaling and pooling earnings for an A equal yields

$$z = \frac{\theta z_{Riley} \pm [\theta z_{Riley} (\theta z_{Riley} - 2\psi) + \psi]^{1/2}}{\psi}.$$

$$\text{Define } z_I = \frac{\theta z_{Riley} + [\theta z_{Riley} (\theta z_{Riley} - 2\psi) + \psi]^{1/2}}{\psi} \text{ and } \hat{z} = \frac{\theta z_{Riley} - [\theta z_{Riley} (\theta z_{Riley} - 2\psi) + \psi]^{1/2}}{\psi}.$$

Numerical values show that As prefer signaling with  $y = y_{Riley} > 1/2$  to pooling if  $\hat{z} < z < z_I$ .

First, I prove that  $\phi \equiv \theta z_{Riley} (\theta z_{Riley} - 2\psi) + \psi > 0$  so the square root of the  $[\bullet]$  term in  $z_I$  and  $\hat{z}$  does not involve imaginary numbers.

Now  $\frac{\partial \phi}{\partial \psi} = 1 - 2\theta z_{Riley} < 0$  since  $z_{Riley} > 1$  and  $\theta > 1$ .

Since  $\psi$  is positively related to  $\theta$ , and the maximum  $\psi = \theta$ , let  $\psi = \theta$  so

$$\phi \equiv \theta [z_{Riley} (\theta z_{Riley} - 2\theta) + 1].$$

Now  $\phi > 0$  if  $\Omega \equiv z_{Riley} (\theta z_{Riley} - 2\theta) + 1 > 0$ .

Since  $\theta z_{Riley} = \theta + [(\theta - 1)\theta]^{1/2}$ , then  $\theta z_{Riley} - 2\theta = [(\theta - 1)\theta]^{1/2} - \theta$ .

Thus,  $\Omega = [(\theta - 1)\theta]^{1/2} - \theta + (\theta - 1)\theta - [(\theta - 1)\theta]^{1/2} + 1 = \theta(\theta - 2) + 1$ .

Now  $\frac{\partial \Omega}{\partial \theta} = 2(\theta - 1)$ , and  $\frac{\partial^2 \Omega}{\partial \theta^2} = 2$ , so  $\Omega$  is minimized when  $\theta = 1$  and  $\Omega = 0$ .

Thus,  $\Omega > 0$  for  $\theta > 1$ , so  $\phi > 0$ .

If  $z_{Riley} < z_I$ ,  $z_I$  is irrelevant since excessive investment in education by As does not occur if  $z > z_{Riley}$ .

Since  $z_I$  is inversely related to  $\psi$ , again let  $\psi = \theta$  (the maximum value for  $\psi$ ), so  $z_I$  is minimized.

Then I find that the condition  $z_{Riley} < z_I$  reduces to  $\phi > 0$ , which has already been proven to be true.

Table One. Critical values for $z$ .				
$\theta$	$\alpha$	$\hat{z}$	$z^*$	$z_{Riley}$
1.5	.2	1.017	.855	1.577
1.5	.4	1.035	1.198	1.577
1.5	.6	1.055	1.382	1.577
1.5	.8	1.077	1.498	1.577
2	.2	1.046	1.076	1.707
2	.4	1.103	1.4	1.707
2	.6	1.18	1.555	1.707
2	.8	1.297	1.647	1.707
3	.2	1.05	1.329	1.816
3	.4	1.113	1.597	1.816
3	.6	1.198	1.712	1.816
3	.8	1.331	1.776	1.816
4	.2	1.052	1.472	1.866
4	.4	1.117	1.622	1.866
4	.6	1.206	1.786	1.866
4	.8	1.345	1.835	1.866

Note, for  $z > \hat{z}$ , As prefer separating at  $y_{Riley} > 1/2$  to pooling.  
For  $z < z_{Riley}$ , As must set  $y = y_{Riley} > 1/2 =$  the perfect information  $y$  for As in order to separate from Bs. Also,  $z^*$  maximizes welfare assuming  $y_{Riley} > 1/2$ .

<b>Table Two. Welfare in separating/signaling equilibria.</b>						
$\theta$	$\alpha$	$z$	$z_{Riley}$	$y_A$	$y_B$	$W$
1.5	.2	1.2	1.577	.657	.417	.234
1.5	.2	1.4	1.577	.563	.357	.217
1.5	.2	1.5	1.577	.526	.333	.208
1.5	.2	1.6	1.577	.5	.313	.231
1.5	.2	1.8	1.577	.5	.278	.214
2	.2	1.2	1.707	.711	.417	.249
2	.2	1.4	1.707	.61	.357	.238
2	.2	1.5	1.707	.569	.333	.231
2	.2	1.6	1.707	.533	.313	.225
2	.2	1.8	1.707	.5	.278	.239
3	.2	1.2	1.816	.757	.417	.277
3	.2	1.4	1.816	.649	.357	.28
3	.2	1.5	1.816	.605	.333	.277
3	.2	1.6	1.816	.568	.313	.272
3	.2	1.8	1.816	.505	.278	.261
1.5	.4	1.2	1.577	.657	.417	.26
1.5	.4	1.4	1.577	.563	.357	.255
1.5	.4	1.5	1.577	.528	.333	.25
1.5	.4	1.6	1.577	.5	.313	.244
1.5	.4	1.8	1.577	.5	.278	.233
2	.4	1.2	1.707	.711	.417	.279
2	.4	1.4	1.707	.61	.357	.297
2	.4	1.5	1.707	.569	.333	.296
2	.4	1.6	1.707	.533	.313	.293
2	.4	1.8	1.707	.5	.278	.283
3	.4	1.2	1.707	.757	.417	.346
3	.4	1.4	1.816	.649	.357	.381
3	.4	1.5	1.816	.605	.333	.387
3	.4	1.6	1.816	.568	.313	.388
3	.4	1.8	1.816	.505	.278	.383

Note, for  $z > z_{Riley}$ , the perfect information level of  $y$  for As, .5, allows separation of As from Bs. If  $z < z_{Riley}$ , As set  $y = y_{Riley} > .5$  (eq.(2)). In all cases, Bs choose their perfect information level of  $y = \frac{1}{2z}$ .

<b>Table Three. Relative education and welfare (lifetime earnings) for As and Bs.</b>			
$\theta$	$z$	$y_A/y_B$	$W_A/W_B$
1.5	1.2	1.576	1.623
1.5	1.5	1.58	2.244
2	1.2	1.705	1.973
2	1.5	1.817	2.868
3	1.2	1.815	2.649
3	1.5	1.817	4.302

$W_A$  denotes welfare for an  $A$ , and  $W_B$  denotes welfare for a  $B$ .

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